Incentives and Endogenous Risk Taking : A Structural View on Hedge Fund Alphas The Journal of Finance

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Motivation

"The superior performance of the financial services sector in the years leading up to the credit crisis was almost entirely due to luck rather than skill and banks increasingly gambled on luck in an effort to keep up with their peers. [...] Good luck and good management need to be better distinguished. "

Norma Cohen(2009), 'Bank profits were due to luck, not skill',Financial Times

- Likely to be even more relevant for hedge funds due to their obvious non-linear incentives.

Hedge funds different in many ways

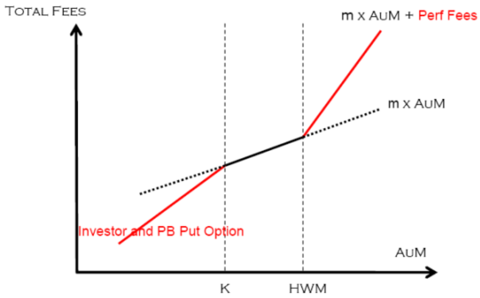
	MFs	*HFs*
Legal	Mgmt Comp/Inv Trust	LP/GP Partnership
*Capital Structure**	No Leverage	PB debt, Fragile
*Positions***	Long	OTC, Short, Deriv.
*Investment Mandate****	Relative return, TE	Absolute return, HWM
Liquidity	Daily	Lockups, Notice per.

* (Liu and Mello (2009), Brunnermeier and Pedersen (2009), Dai and Sundaresan (2010))
** (Almazan, Brown, Carlson and Chapman (2004))
*** (Agarwal, Daniel and Naik (2009), Aragon and Nanda (2012))

Intuition of Manager's Investment Problem

We include and study the implications of **funding** and **redemption** options (see also Koijen (2010), and Dai and Sundaresan (2010))

$$\max_{(\theta_s)_{s\in[0,T]}} E_0[U(p(X_T - HWM)^+ + mX_T - c(K - X_T)^+)] \qquad (1)$$



where p denotes performance fee, m denotes management fee and c denotes the level of concern regarding the short put option positions.

Outline

✓ Motivation

- Main Findings
- Related Literature
- Structural Model (Solution, Calibration)
- Data
- Empirical Evidence on Risk Shifting
- In-sample Estimation of the Model
- Out-of-sample Test

Main Findings

- Optimal portfolio choice of a hedge fund manager differs from classical Merton solution
  - depends on fund value relative to high-water mark and put option's strike price
- Perf Fees encourages a manager to take more risk; short put options moderate the effect
- Reduced-form alpha mixture of true skill and endogenous incentives of the manager.
- We document the risk-shifting in hedge funds
  1. Advantage of structural apporach: disentangles true managerial skills from risk tolerance in hedge funds
  2. Advantage of structural approach: Improvement in efficiency of skill estimator since informatin in time-varying second moments used
  3. Structural alpha generates superior out-of-sample performance

Literature Review and Contribution

Literature	Objective function	Optimal Allocation
Merton(1969), Karatzas(1987)	Terminal wealth	$\theta = \frac{\mu - r}{\gamma\sigma^2}$
Carpenter(2000)	Call option on terminal wealth	$\theta_t \rightarrow \infty$ as $X_t \rightarrow 0$ and $\theta_t \rightarrow \frac{\mu - r}{\gamma\sigma^2}$ as $X_t \rightarrow \infty$
Panageas & Westerfield(2009) ,Guasoni & Obloj(2010)	Perf fee + Mgmt fee (Infinite horizon)	$\theta = \frac{1}{1-\eta}\frac{\mu - r}{\sigma^2}$
Hodder and Jackwerth(2007)	Perf fee + Mgmt fee + Liquidation boundary (Finite horizon)	Similar to Carpenter(2000) but manager scales down risk as $X_t \rightarrow 0$
Our paper	Perf fee + Mgmt fee + Funding & Redemption options (Finite horizon)	Similar to Hodder & Jackwerth(2007) but manager more aggressively scales down risk as $X_t \rightarrow K$

- Money market account with a constant interest rate r:

$$dS_t^0 = S_t^0 r dt. \tag{2}$$

- Benchmark asset S_t^B , which follows

$$dS_t^B = S_t^B(r + \sigma_B\lambda_B)dt + S_t^B\sigma_B dZ_t^B. \tag{3}$$

- Alpha technology S_t^A , which follows

$$dS_t^A = S_t^A(r + \alpha^*)dt + S_t^A\sigma_A dZ_t^A, \tag{4}$$

where $\alpha^* = \sigma_A\lambda_A$. λ_A is a proxy of true managerial skill (true Sharpe ratio of the fund).

- The manager allocates portfolio θ_t to invest in the investment opportunity set.

$$dX_t = X_t(r + \theta_t^{'}\mu)dt + X_t\theta_t^{'}\Sigma dZ_t \tag{5}$$

where $\mu \equiv (\alpha^*, \sigma_B\lambda_B)', \Sigma \equiv diag(\sigma_A, \sigma_B)$ and $Z_t = (Z_t^A, Z_t^B)'$

The Model - Solution

- Use martingale approach (as in Cox and Huang (1989))
- Assuming markets are complete and state price process φ_t :

$$\frac{d\varphi_t}{\varphi_t} = -rdt - \lambda^{'} dZ_t, \tag{6}$$

- the solution of the optimal investment problem solves:

$$\max_{X_T} E_0 [U(p(X_T - HWM)^+ + mX_T - c(K - X_T)^+)] \tag{7}$$

subject to

$$E_0[\varphi_T X_T] = X_0 \tag{8}$$

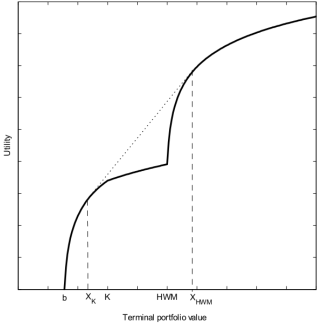
where $U(W)$ is CRRA of the form $U(W) = \frac{W^{1-\gamma}}{1-\gamma}$.

- To solve the problem, we use the **concavification techniques** discussed and used in Carpenter (2000) and Basak et al. (2007)

Concavification

- Since the two options have different strike prices, the problem is in general not concave.
- Proceed using standard concavification techniques:

Figure 2: Concavified Utility Function. This figure displays the concavified utility function at different levels of terminal fund value. The solid line is the original utility function which is superimposed with a dotted line between X_K and X_{HWM}.



- The optimal allocation :

$$\theta_t^* = -\frac{\partial X^*(t, \varphi_t)}{\partial \varphi_t} \frac{\varphi_t}{X_t^*} (\Sigma\Sigma')^{-1} \Sigma \lambda \tag{9}$$

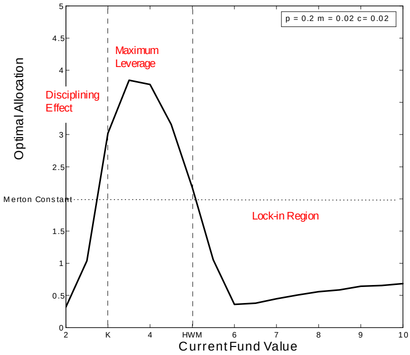
- Draw an analogy to Merton (1969) solution and rewrite the above (see paper) as:

$$\theta_t^* = \frac{(\Sigma\Sigma')^{-1} \Sigma \lambda}{\widetilde{\gamma}_t}$$

where $\widetilde{\gamma}_t(\gamma, \lambda, X_t, HWM, K, p, m, c) \equiv -\frac{\partial X^*(t,\varphi_t)}{\partial \varphi_t} \frac{\varphi_t}{X_t^*}$.

- Even if drift and diffusion terms of the investment opportunity set are constant, the optimal allocation is time varying and state dependent

The Model - Solution (Figure 3)



- *Lock-in*: When fund value exceeds the high-water mark level, the manager deleverages below Merton (1969) solution. See also (Hodder and Jackwerth, and Carpenter)
- *Short put option*: Below a threshold, it is optimal to deleverage aggressively because the manager has more to lose than to gain.

Cross-Restrictions

A standard performance regression:

$$\frac{dX_t}{X_t} - rdt = \hat{\alpha}_r dt + \hat{\beta}_r (\frac{dS_t^B}{S_t^B} - rdt) + \hat{\sigma}_{\varepsilon,r} dZ_t^A. \tag{10}$$

However, NAV_t process affected endogenously by the dynamics of θ_t^*:

$$\frac{dX_t^*}{X_t^*} = (r + \frac{\alpha^{*2}}{\widetilde{\gamma}_t \sigma_A^2} + \frac{1}{\widetilde{\gamma}_t} \lambda_B^2) dt + \frac{\lambda_B}{\widetilde{\gamma}_t} dZ_t^B + \frac{\alpha^*}{\widetilde{\gamma}_t \sigma_A} dZ_t^A. \tag{11}$$

Implications of structural model:

$$H1 \quad : \quad \hat{\alpha}_{t,OLS} = \frac{\alpha^{*2}}{\widetilde{\gamma}_t \sigma_A^2} = \frac{\lambda_A^2}{\widetilde{\gamma}_t}$$

$$H2 \quad : \quad \hat{\beta}_{t,OLS} = \frac{\lambda_B}{\widetilde{\gamma}_t \sigma_B},$$

$$\textit{and} \qquad \hat{\sigma}_{t,OLS,\varepsilon} = \frac{\alpha^*}{\widetilde{\gamma}_t \sigma_A} = \frac{\lambda_A}{\widetilde{\gamma}_t}.$$

where $\widetilde{\gamma}_t \equiv f(\gamma, \lambda, X_t, HWM, K, p, m, c)$

Koijen (2010) studies similar set of restrictions in mutual funds.

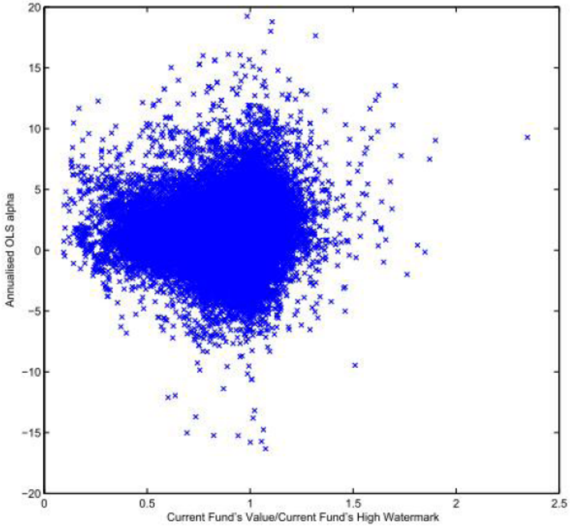
Important insight from structural model

- The reduced-form alpha is a mixture of true managerial skill and endogenous incentives $\tilde{\gamma}_t$:

$$\hat{\alpha}_{t,OLS} = \theta^*_{At}\alpha^* = \frac{\alpha^{*2}}{\widetilde{\gamma}_t \sigma_A^2}$$

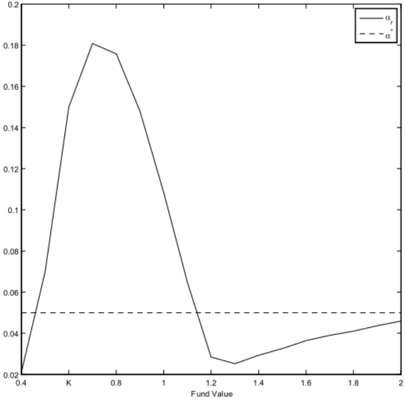
- If optimal portfolio choice is Merton's type, then reduced-form alpha is unbiased.
- However, if optimal portfolio is state-dependent, then there is a bias in reduced-form alpha.
- Nonlinear contracts in hedge funds create natural incentives for a state-dependent optimal portfolio
- Estimate $\widehat{\alpha}_{t,OLS}$ using reduced form regression with constant coefficients is misspecified.

Scatter Plot of Fung and Hsieh Alpha - Figure 5



Reduced-Form and True Alpha: Calibration

- "Assume access to alpha technology with $\alpha^* = 0.5$. What would α_{OLS} say?"



Reduced-Form and Structural Skill Estimates in Simulated Economies Table 1

True parameter	Panel A: Reduced-form Estimation		
λ_A	$\widehat{\alpha}_{OLS}$	$\widehat{\beta}_{OLS}$	IR
0.5	5.93	1.82	0.29
	(10.15)	(0.74)	(065)
1	19.67	2.04	0.85
	(14.99)	(0.92)	(0.65)
1.5	39.82	2.14	1.37
	(18.20)	(1.11)	(0.64)

True parameter	Panel B: Structural Estimation				
λ_A	$\widehat{\lambda}_A$	Mean ($\widehat{\alpha}_{t,OLS}$)	Std ($\widehat{\alpha}_{t,OLS}$)	Mean ($\widehat{\beta}_{t,OLS}$)	Std ($\widehat{\beta}_{t,OLS}$)
0.5	0.49	5.39	3.12	1.76	1.26
	(0.09)	(2.44)	(1.49)	(0.57)	(0.50)
1	0.99	19.12	8.05	2.03	1.60
	(0.15)	(5.95)	(2.60)	(0.58)	(0.41)
1.5	1.48	38.80	13.68	2.12	1.83
	(0.19)	(9.28)	(3.13)	(0.52)	(0.31)

Out-of-sample analysis - Simulated Economy: Table 2

- Assume 10% funds have $\alpha^* = 5\%$ and others have $\alpha^* = 0$.
- Compute Rolling EW Top-Dec portfolio

Panel A: Out-of-sample performance metrics

Portfolio	Alpha (pct/ann.)	*t*-stat	Mean Ret (pct/ann.)	\$1 growth	IR	SR
OLS Alpha	1.96	1.77	7.80	2.08	0.56	0.30
Structural True Skill	6.27	6.81	12.09	3.20	2.16	0.78

Data

- Hedge funds monthly net-of-fee returns between January 1994 and December 2010 from BarclayHedge
- Advantages of BarclayHedge data base (Joenvaara, Kosowski and Tolonen , 2012)
- Consider only funds with minimum 36 monthly returns observations
- There are 4954 funds across 11 strategies

Data

Table 4: Summary Statistics for Hedge Funds Returns. This table displays, for each invesment category, the number of funds (in parentheses) and fund cross-sectional median as well as first and third quartiles (in parentheses) of the annualised excess mean returns over risk free rate, standard deviation, skewness and kurtosis. The statistics are computed using monthly net-of-fee returns of live and dead hedge funds reported in the BarclayHedge database between January 1994 and December 2010.

Investment Objectives	Mean (pct/ann.)	Std (pct/ann.)	Skewness	Kurtosis
CTA (1655)	5.80 (1.33 , 12.06)	15.28 (9.94 , 24.57)	0.34 (-0.12 , 0.86)	4.28 (3.41 , 6.03)
Convertible Arbitrage (155)	4.97 (2.34 , 6.49)	6.98 (4.22 , 10.55)	-0.72 (-1.45 , -0.08)	6.13 (4.37 , 10.97)
Emerging Markets (512)	9.20 (3.98 , 16.04)	19.40 (10.97 , 29.33)	-0.30 (-0.94 , 0.25)	5.14 (3.84, 7.92)
Equity Long/Short(807)	6.57 (2.74 , 11.21)	11.73 (8.43 , 17.54)	-0.01 (-0.53 , 0.54)	4.52 (3.56 , 6.24)
Equity Market Neutral (154)	2.64 (-0.11 , 5.64)	7.27 (5.28 , 10.13)	-0.10 (-0.42 , 0.22)	4.14 (3.10 , 5.58)
Equity Short Bias (34)	1.40 (-3.76 , 6.40)	21.57 (13.40 , 30.79)	0.30 (-0.08 , 0.66)	4.32 (3.56 , 5.46)
Event Driven (211)	7.10 (4.34 , 10.65)	10.84 (7.19 , 14.86)	-0.43 (-1.19 , 0.32)	5.91 (4.53 , 9.79)
Fixed Income Arbitrage (93)	3.57 (0.31 , 8.53)	9.02 (5.52 , 12.99)	-0.77 (-2.45 , 0.06)	8.53 (4.68 , 15.20)
Global Macro (185)	5.79 (2.55 , 10.43)	12.35 (8.58 , 17.08)	0.09 (-0.43 , 0.49)	4.33 (3.56 , 6.24)
Multi Strategy (196)	5.23 (1.22 , 8.96)	9.85 (6.04 , 15.13)	-0.38 (-1.21 , 0.34)	5.35 (3.60 , 9.36)
Others (826)	8.23 (3.66 , 14.15)	13.91 (8.44 , 23.29)	-0.03 (-0.65 , 0.59)	4.90 (3.60 , 7.00)
All Funds (4828)	6.38 (2.26 , 11.99)	13.33 (8.29 , 21.68)	0.05 (-0.56 , 0.62)	4.70 (3.58 , 6.94)

Structural Estimation

- First, we estimate the set of a benchmark asset parameters $\hat{\Theta}_B \equiv \{\hat{\sigma}_B, \hat{\lambda}_B\}$.
- Second, Since

$$\frac{dX_t^*}{X_t^*} = (r + \frac{\alpha^{*2}}{\widetilde{\gamma}_t \sigma_A^2} + \frac{1}{\widetilde{\gamma}_t}\hat{\lambda}_B^2)dt + + \frac{1}{\widetilde{\gamma}_t}\hat{\lambda}_B dZ_t^B + \frac{\alpha^*}{\widetilde{\gamma}_t \sigma_A} dZ_t^A, \qquad (12)$$

where $\widetilde{\gamma}_t \equiv f(\gamma, \lambda, X_t, HWM, K, p, m, c)$
we can derive the log-likelihood (assuming log-normality)

$$\arg\max_{\Theta_C} \sum_{t=h}^{T/h} \ell(r_t^X \mid r_t^B; \lambda_A, \gamma, \hat{\Theta}_B)$$

$\Theta_C \equiv \{\lambda_A, \gamma\}$ where $\lambda_A \equiv \frac{\alpha^*}{\sigma_A}$
Using not only information on 1st moment of return but also 2nd moment and high-water mark to draw inference about skill and risk preference.

In-Sample Estimation: Table 9

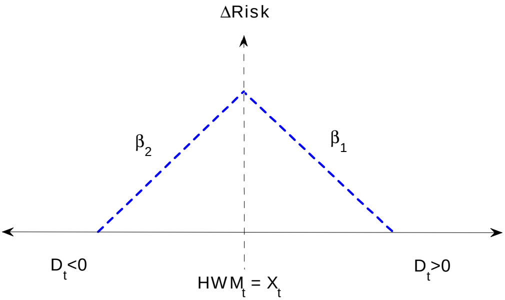
Investment Objectives	True Skill λ_A	OLS Alpha α_{OLS}	Mean Structural Alpha Mean($\alpha_{t,OLS}$)	Std Structural Alpha Std($\alpha_{t,OLS}$)
CTA (1655)	0.55 (0.33, 0.93)	5.58 (1.48 , 11.60)	3.14 (1.57, 6.53)	2.16 (0.93, 4.24)
Convertible Arbitrage (155)	0.00 (0.00, 0.26)	3.19 (1.03 , 5.60)	0.00 (0.00, 4.58)	0.00 (0.00, 0.56)
Emerging Markets (512)	0.70 (0.37, 1.01)	6.09 (1.60 , 12.07)	3.29 (1.03, 7.46)	3.96 (1.18, 7.07)
Equity Long/Short(807)	0.44 (0.31, 0.67)	4.34 (0.82 , 8.44)	1.23 (0.59, 2.75)	1.24 (0.49, 2.54)
Equity Market Neutral (154)	0.12 (0.00, 0.26)	2.00 (-0.99 , 4.53)	1.06 (0.00, 1.86)	0.40 (0.00, 1.08)
Equity Short Bias (34)	0.47 (0.25, 1.15)	3.20 (0.02 , 9.18)	6.90 (2.95, 15.77)	2.82 (1.46, 7.61)
Event Driven (211)	0.41 (0.30, 0.59)	4.11 (0.58 , 8.34)	1.11 (0.45, 2.20)	1.31 (0.12, 2.86)
Fixed Income Arbitrage (93)	0.17 (0.10, 0.33)	1.63 (-2.08 , 5.26)	0.65 (0.27, 1.31)	0.51 (0.18, 1.42)
Global Macro (185)	0.61 (0.46, 0.80)	4.33 (1.13 , 8.83)	1.89 (1.14, 2.82)	1.56 (0.75, 2.55)
Multi Strategy (196)	0.41 (0.04, 0.74)	5.99 (3.39 , 9.20)	6.37 (0.66, 11.40)	2.22 (0.20, 4.82)
Others (826)	0.53 (0.34, 0.88)	5.15 (0.74 , 10.44)	1.58 (0.51, 4.87)	1.19 (0.12, 4.76)
All Funds (4828)	0.49 (0.29, 0.82)	4.85 (1.09, 9.87)	2.08 (0.75, 5.24)	1.62 (0.44, 3.96)

Empirical Evidence on Risk Shifting - Piecewise Regression

- Risk shifting?

$$\Delta\sigma_{i,[t,t+12]} = u_0 + 1_{\{D_{i,t}>0\}}\beta_1\mathcal{D}_{i,t} + 1_{\{D_{i,t}<0\}}\beta_2\mathcal{D}_{i,t} + \beta_3 \times Controls + \varepsilon_i$$
$$\text{where } \mathcal{D}_{i,t} \equiv X_{i,t}/HWM_{i,t} - 1$$

Panageas and Westerfield (2009):	$H_0 : \beta_1 = \beta_2 = 0$
Carpenter (2000):	$H_0 : \beta_1 < 0$, and $\beta_2 < 0$
... this paper (BKW 2013):	$H_0 : \beta_1 < 0$, and $\beta_2 > 0$



Empirical Evidence on Risk Shifting: Table 11

$$\sigma_{i,[t,t+T]} - \sigma_{i,[t-T,t]} = u_i + \beta_1 \times 1_{\{Dist2HWM_{i,t}>0\}} \times Dist2HWM_{i,t} + \beta_2 \times 1_{\{Dist2HWM_{i,t}<0\}} \times Dist2HWM_{i,t} + \beta_3 \times (AVGVIX_{[t,t+T]} - AVGVIX_{[t-lag,t]}) + \varepsilon_{i,t},$$

	Panel A : $\sigma_{t+12} - \sigma_{t-12}$					Panel B: $\sigma_{t+6} - \sigma_{t-6}$				
Investment Objectives	Intercept	$Beta1$	$Beta2$	$Beta3$	$Adj.R^2$	Intercept	$Beta1$	$Beta2$	$Beta3$	$Adj.R^2$
All Strategies	0.009***	-0.063***	0.137***	0.556***	0.188	-0.004***	-0.038***	0.087***	0.528***	0.115
CTA	0.001	-0.096***	0.025***	0.304***	0.104	-0.010***	-0.067***	-0.057***	0.303***	0.058
Convertible Arbitrage	0.009***	-0.067***	0.092***	1.116***	0.404	-0.001***	-0.078***	0.124***	0.997***	0.319
Emerging Markets	0.006***	-0.015**	0.127***	0.913***	0.281	-0.012***	0.020***	0.098***	0.839***	0.198
Equity Long/Short	0.005***	-0.019***	0.118***	0.408***	0.126	-0.005***	-0.005	0.085***	0.412***	0.088
Equity Market Neutral	-0.003***	0.056***	0.029**	0.224***	0.181	-0.009***	0.067***	-0.009	0.208***	0.111
Equity Short Bias	0.052***	-0.156***	0.196***	0.418***	0.068	0.009	-0.120**	0.028	0.620***	0.040
Event Driven	0.008***	-0.101***	0.101***	0.782***	0.327	0.001	-0.108***	0.127***	0.706***	0.266
Fixed Income Arbitrage	0.019***	-0.173***	0.339***	0.833***	0.434	0.014***	-0.195***	0.377***	0.758***	0.348
Global Macro	0.004**	-0.054***	0.036**	0.457***	0.137	-0.011***	0.027**	-0.054***	0.411***	0.097
Multi Strategy	0.009***	-0.065***	0.167***	0.669***	0.434	0.003***	-0.069***	0.206***	0.604***	0.338
Others	0.014***	-0.040***	0.185***	0.642***	0.284	0.000	-0.016**	0.149***	0.631***	0.159

Sorted by skill

$$\mathcal{R}_{i,t} = u + c_1 \times 1_{\{Dist2HWM_{i,t}>0\}} \times Dist2HWM_{i,t} + c_2 \times 1_{\{Dist2HWM_{i,t}<0\}} \times Dist2HWM_{i,t} + \varepsilon_{i,t},$$

	Intercept	c_1	c_2	$Adj.R^2$
Panel A: $\mathcal{R}_{i,t} \equiv$ Standardized Beta				
All funds	0.86	-0.18	0.24	0.014
	(0.00)	(0.00)	(0.01)	
Low skill funds	0.90	-0.30	0.43	0.003
	(0.00)	(0.02)	(0.02)	
High skill funds	0.89	-0.11	0.24	0.003
	(0.00)	(0.20)	(0.03)	
Panel B: $\mathcal{R}_{i,t} \equiv$ Standardized Residual Volatility				
All funds	0.08	-0.01	0.01	0.001
	(0.00)	(0.13)	(0.21)	
Low skill funds	0.07	-0.04	0.09	0.02
	(0.00)	(0.00)	(0.00)	
High skill funds	0.13	-0.03	0.03	0.001
	(0.00)	(0.10)	(0.25)	

Out-of-Sample Performance

- True skill measure estimate is a superior predictor of performance since it distinguishes skill from risk taking
- We form top-quartile portfolios ranked by true skill and reduced form skill measure
- The portfolios are rebalanced every January based on 36 months returns.
- The portfolios are between January 1997 and December 2010

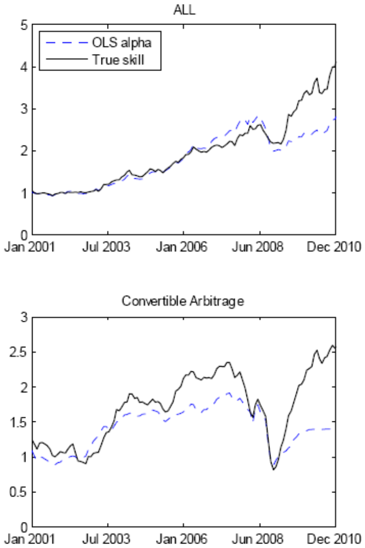
Out-of-Sample Performance - Ranking funds on OLS Alphas : Table 13

Portfolio	FH Alpha (pct/ann.)	*t*-stat	*p*-value	Adj R^2	Mean Ret (pct/ann.)	\$1 growth	IR	TE	SR
Decile 1	6.28	2.13	0.04	40.37	11.25	2.90	0.78	8.00	0.85
Decile 2	3.14	1.77	0.08	40.43	7.03	1.96	0.58	5.45	0.66
Decile 3	3.36	2.21	0.03	33.04	7.44	2.07	0.77	4.36	0.95
Decile 4	3.46	3.18	0.00	40.37	6.72	1.93	0.92	3.77	0.90
Decile 5	3.38	2.86	0.01	42.46	7.15	2.02	1.01	3.34	1.09
Decile 6	2.79	3.00	0.00	49.97	6.30	1.86	0.94	2.98	0.95
Decile 7	2.40	2.58	0.01	47.21	6.01	1.81	0.85	2.81	0.95
Decile 8	2.79	3.54	0.00	47.55	6.13	1.83	1.05	2.65	1.05
Decile 9	3.23	2.82	0.01	52.86	6.69	1.93	1.03	3.14	0.96
Decile 10	4.43	2.71	0.01	56.30	8.05	2.18	1.00	4.44	0.85
Spread (Decile1-10)	-0.46	-0.14	0.89	12.83	3.20	1.33	-0.06	7.41	0.12

Out-of-Sample Performance - Ranking funds on True Skills : Table 14

Portfolio	Alpha (pct/ann.)	*t*-stat	*p*-value	Adj R^2	Mean Ret (pct/ann.)	\$1 growth	IR	TE	SR
Decile 1	11.44	3.55	0.00	41.73	15.50	4.32	1.22	9.41	1.05
Decile 2	5.58	3.15	0.00	52.14	9.36	2.45	0.98	5.69	0.85
Decile 3	5.09	2.89	0.00	42.92	9.34	2.47	0.95	5.38	0.98
Decile 4	2.25	1.36	0.18	42.24	6.85	1.95	0.51	4.37	0.78
Decile 5	2.87	2.35	0.02	43.72	6.56	1.90	0.76	3.79	0.83
Decile 6	2.21	1.89	0.06	41.86	5.96	1.79	0.64	3.43	0.81
Decile 7	2.35	2.50	0.01	47.30	5.76	1.76	0.86	2.75	0.91
Decile 8	1.45	1.63	0.11	48.58	4.91	1.62	0.56	2.60	0.72
Decile 9	1.58	2.32	0.02	27.06	4.65	1.58	0.73	2.16	0.91
Decile 10	0.43	0.38	0.71	39.20	3.87	1.46	0.19	2.25	0.54
Spread (Decile1-10)	8.70	2.43	0.02	35.29	11.64	2.98	0.94	9.26	0.80

OOS Results - Figure 9



Summary

- Optimal portfolio choice of a hedge fund manager differs from classical Merton solution
  - depends on fund value relative to high-water mark and put option's strike price
- High-water mark encourages a manager to take more risk while short put options moderate the effect
- Reduced-form alpha mixture of true skill and risk preference of the manager.
- We document the risk-shifting in hedge funds
- The model disentangles true managerial skills from risk tolerance in hedge funds; structural alpha generates superior out-of-sample performance

THANK YOU

- The optimal allocation :

$$\theta_t^* = -\frac{\partial X^*(t,\varphi_t)}{\partial \varphi_t}\frac{\varphi_t}{X_t^*}(\Sigma\Sigma')^{-1}\Sigma\lambda \tag{13}$$

where

$$\begin{aligned}
\frac{\partial X^*(t,\varphi_t)}{\partial \varphi} = & -\frac{\zeta^{-1/\gamma}G_t}{\varphi_t^{\frac{1+\gamma}{\gamma}}(p+m)^{1-1/\gamma}}\left(\frac{N(d_{2,t}^{z_{HWM}})}{\gamma}+\frac{N'(d_{2,t}^{z_{HWM}})}{\|\lambda\|\sqrt{T-t}}\right)-\frac{\alpha HWM}{\varphi_t(\alpha+m)}\frac{H_tN'(d_{1,t}^{z_{HWM}})}{\|\lambda\|\sqrt{T-t}} \qquad (12)\\
& -\frac{\widehat{X}_{HWM}H_t}{\varphi_t\|\lambda\|\sqrt{T-t}}\left[N'(d_{1,t}^{z_K})-N'(d_{1,t}^{z_{HWM}})\right]\\
& -\frac{\zeta^{-1/\gamma}G_t}{\varphi_t^{\frac{1+\gamma}{\gamma}}(m+c)^{1-1/\gamma}}\left[\frac{N'(d_{2,t}^{z_b})-N'(d_{2,t}^{z_K})}{\|\lambda\|\sqrt{T-t}}+\frac{N(d_{2,t}^{z_b})-N(d_{2,t}^{z_K})}{\gamma}\right]\\
& -\frac{cKH_t}{\varphi_t(m+c)\|\lambda\|\sqrt{T-t}}(N'(d_{1,t}^{z_b})-N'(d_{1,t}^{z_K}))+b\frac{H_tN(-d_{1,t}^{z_b})}{\varphi_t\|\lambda\|\sqrt{T-t}} \qquad (13)
\end{aligned}$$