Internet Appendix for "Predictive Regressions: A Present-Value Approach"*

In this appendix we provide several additional derivations, results and tables. In Section IA.A we provide the derivations of the present-value model. In Section IA.B we analyze the finite-sample properties of our maximum likelihood estimators. In Section IA.C we discuss the reinvestment strategy and how it relates to the model specification for dividend growth. In Section IA.D we provide the likelihood ratio statistics, and in Section IA.E we provide our estimates for the sample period 1927 to 2007.

IA.A. Derivation of the Present-value Model

We consider the model

$$\begin{aligned}
\Delta d_{t+1} &= g_t + \varepsilon^d_{t+1}, \\
g_{t+1} &= \gamma_0 + \gamma_1 (g_t - \gamma_0) + \varepsilon^g_{t+1}, \\
\mu_{t+1} &= \delta_0 + \delta_1 (\mu_t - \delta_0) + \varepsilon^\mu_{t+1},
\end{aligned}$$

where

$$\begin{aligned}
\Delta d_{t+1} &\equiv \log\left(\frac{D_{t+1}}{D_t}\right), \\
\mu_t &\equiv E_t[r_{t+1}], \\
r_{t+1} &\equiv \log\left(\frac{P_{t+1}+D_{t+1}}{P_t}\right).
\end{aligned}$$

We also define $pd_t = \log(PD_t)$. Now consider the log-linearized return, with $\overline{pd} = E\left[pd_t\right]$:

$$\begin{aligned}
r_{t+1} &= \log\left(1+\exp\left(pd_{t+1}\right)\right) + \Delta d_{t+1} - pd_t \\
&\simeq \log\left(1+\exp\left(\overline{pd}\right)\right) + \frac{\exp\left(\overline{pd}\right)}{1+\exp\left(\overline{pd}\right)}\left(pd_{t+1} - \overline{pd}\right) + \Delta d_{t+1} - pd_t \\
&= \kappa + \rho pd_{t+1} + \Delta d_{t+1} - pd_t.
\end{aligned}$$

Equivalently, we have

$$pd_t = \kappa + \rho pd_{t+1} + \Delta d_{t+1} - r_{t+1},$$

where

$$\begin{aligned}
\kappa &= \log\left(1+\exp\left(\overline{pd}\right)\right) - \rho\overline{pd}, \\
\rho &= \frac{\exp\left(\overline{pd}\right)}{1+\exp\left(\overline{pd}\right)}.
\end{aligned}$$

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By iterating this equation we find

$$
\begin{aligned}
pd_t &= \kappa + \rho pd_{t+1} + \Delta d_{t+1} - r_{t+1} \\
&= \kappa + \rho\left(\kappa + \rho pd_{t+2} + \Delta d_{t+2} - r_{t+2}\right) + \Delta d_{t+1} - r_{t+1} \\
&= \kappa + \rho\kappa + \rho^2 pd_{t+2} + \Delta d_{t+1} - r_{t+1} + \rho\left(\Delta d_{t+2} - r_{t+2}\right) \\
&= \sum_{j=0}^{\infty} \rho^j \kappa + \rho^{\infty} pd_{\infty} + \sum_{j=1}^{\infty} \rho^{j-1}\left(\Delta d_{t+j} - r_{t+j}\right) \\
&= \frac{\kappa}{1-\rho} + \sum_{j=1}^{\infty} \rho^{j-1}\left(\Delta d_{t+j} - r_{t+j}\right),
\end{aligned}
$$

assuming that $\rho^{\infty} pd_{\infty} = \lim_{j\to\infty} \rho^j pd_{t+j} = 0$ (in expectation would suffice for our purpose). Next, we take expectations conditional upon time t:

$$
\begin{aligned}
pd_t &= \frac{\kappa}{1-\rho} + \sum_{j=1}^{\infty} \rho^{j-1} E_t\left[\Delta d_{t+j} - r_{t+j}\right] \\
&= \frac{\kappa}{1-\rho} + \sum_{j=1}^{\infty} \rho^{j-1} E_t\left[g_{t+j-1} - \mu_{t+j-1}\right] \\
&= \frac{\kappa}{1-\rho} + \sum_{j=0}^{\infty} \rho^{j} E_t\left[g_{t+j} - \mu_{t+j}\right], \\
&= \frac{\kappa}{1-\rho} + \sum_{j=0}^{\infty} \rho^{j}\left(\gamma_0 + \gamma_1^j\left(g_t - \gamma_0\right) - \delta_0 - \delta_1^j\left(\mu_t - \delta_0\right)\right) \\
&= \frac{\kappa}{1-\rho} + \frac{\gamma_0 - \delta_0}{1-\rho} + \sum_{j=0}^{\infty} \rho^{j}\left(\gamma_1^j\left(g_t - \gamma_0\right) - \delta_1^j\left(\mu_t - \delta_0\right)\right) \\
&= \frac{\kappa}{1-\rho} + \frac{\gamma_0 - \delta_0}{1-\rho} + \frac{g_t - \gamma_0}{1-\rho\gamma_1} - \frac{\mu_t - \delta_0}{1-\rho\delta_1},
\end{aligned}
$$

which uses

$$E_t\left[x_{t+j}\right] = \alpha_0 + \alpha_1^j\left(x_t - \alpha_0\right),$$

provided that

$$x_{t+1} = \alpha_0 + \alpha_1\left(x_t - \alpha_0\right) + \varepsilon_{t+1}.$$

IA.B. Finite-sample Properties

In this section, we analyze the finite-sample properties of our maximum likelihood estimators. We focus on the model for cash-reinvested dividends in Section I. We simulate 1,000 samples with the same number of observations as in the data, starting with a draw from the unconditional distribution of the state variables. We use the point estimates of the second column of Table II as the true parameters in the simulation. We subsequently estimate the model for each of the simulated samples. Table IA.I reports the true parameters along with the average, standard deviation, and quantiles of the distribution of 1,000 parameter estimates in Panel A. Panel B reports the correlation between the parameter estimates.

Panel A shows that δ_1 is somewhat downward biased, while γ_1 is upward biased. This corresponds to an upward bias in σ_μ and a downward bias in σ_g. Further, it appears that the correlation between expected returns and unexpected growth rates, $\rho_{\mu d}$, is not estimated precisely. Panel B shows that the estimates for the persistence of expected returns (δ_1) and the persistence of expected growth rates (γ_1) are negatively correlated. Also, we find the persistence parameters and the conditional volatility parameters to be negatively correlated (e.g., δ_1 and σ_μ).

[Table IA.I about here]

IA.C. Reinvestment Strategy and Model Specification

In the main article, we assume that the conditional expected dividend growth rate is an AR(1) process if dividends are reinvested in the risk-free rate. We subsequently derive the implied dynamics for market-reinvested dividends. We stress again that there is a present-value model for each reinvestment strategy of dividends, reflected in the time-series properties of expected returns and expected dividend growth rates. We now consider the present-value model in equations (4) to (6), but apply it to market-reinvested dividends instead of cash-reinvested dividends. To be sure, we consider this model to be misspecified, but we wish to explore its characteristics to better understand the link between dividend reinvestment strategies. We thus estimate an alternative specification in which expected growth rates of *market-reinvested* dividends are modeled as an AR(1) process. The parameter estimates of this model are presented in Table IA.II. The table shows that the estimated value of γ_1 is not only lower than in the model in which cash-reinvested expected dividend growth is an AR(1) process, but it is in fact estimated to be *negative*. Despite this negative value for γ_1, we still find relatively high R^2 values for both returns and dividend growth rates.

[Table IA.II about here]

To further explore this evidence of a negative estimated value for γ_1 in this model, we construct a grid of possible levels of γ_1. For each point in the grid, we optimize over the other parameters and record the associated likelihoods and parameter estimates, as shown in Table IA.III. The main results are summarized in Panel A of Figure IA.1, where we plot the likelihood as a function of γ_1. The picture shows that the likelihood has two

Table IA.I. Finite-sample properties of the maximum-likelihood estimators.

The table contains results about the finite-sample properties of our maximum-likelihood estimators. We focus on the model for cash-reinvested dividends in Section I. We simulate 1,000 samples with the same number of observations as in the data, starting with a draw from the unconditional distribution of the state variables. We use the point estimates of the second column of Table II in the simulation. We subsequently estimate the model for each of the simulated samples. Panel A reports the true parameters along with the average, standard deviation, and quantiles of distribution of 1,000 parameter estimates. Panel B depicts the correlation between the parameter estimates.

Panel A: Mean, standard deviation, and quantiles

	True	Average	St.dev.	Q(0.10)	Q(0.25)	Q(0.50)	Q(0.75)	Q(0.90)
δ_0	0.090	0.090	0.020	0.067	0.077	0.089	0.101	0.113
δ_1	0.932	0.864	0.128	0.765	0.837	0.887	0.926	0.952
γ_0	0.062	0.061	0.011	0.047	0.054	0.061	0.069	0.076
γ_1	0.354	0.429	0.271	0.218	0.304	0.417	0.565	0.764
σ_μ	0.016	0.025	0.013	0.012	0.016	0.022	0.030	0.041
σ_g	0.058	0.045	0.017	0.017	0.036	0.052	0.057	0.061
σ_d	0.002	0.022	0.019	0.003	0.006	0.014	0.040	0.051
$\rho_{g\mu}$	0.417	0.318	0.375	-0.009	0.254	0.403	0.516	0.605
$\rho_{\mu d}$	-0.147	0.176	0.579	-0.808	-0.180	0.298	0.640	0.860
A	3.612	3.546	0.421	3.135	3.345	3.551	3.771	3.979
B_1	13.484	8.009	4.088	3.870	5.288	7.116	9.709	12.891
B_2	2.616	2.281	2.001	1.268	1.418	1.678	2.212	3.855

Panel B: Correlation matrix

	δ_0	δ_1	γ_0	γ_1	σ_μ	σ_g	σ_d	$\rho_{g\mu}$	$\rho_{\mu d}$
δ_0	1.000	0.008	0.783	0.063	-0.021	0.011	-0.022	-0.021	-0.015
δ_1	0.008	1.000	0.007	-0.175	-0.686	0.235	-0.189	0.061	-0.149
γ_0	0.783	0.007	1.000	0.067	-0.024	0.012	-0.029	-0.034	-0.034
γ_1	0.063	-0.175	0.067	1.000	0.107	-0.280	0.337	0.152	0.072
σ_μ	-0.021	-0.686	-0.024	0.107	1.000	-0.105	0.104	0.105	0.233
σ_g	0.011	0.235	0.012	-0.280	-0.105	1.000	-0.885	0.496	-0.167
σ_d	-0.022	-0.189	-0.029	0.337	0.104	-0.885	1.000	-0.402	0.194
$\rho_{g\mu}$	-0.021	0.061	-0.034	0.152	0.105	0.496	-0.402	1.000	-0.072
$\rho_{\mu d}$	-0.015	-0.149	-0.034	0.072	0.233	-0.167	0.194	-0.072	1.000

Table IA.II. Estimation results of the model in (4)-(6) using market-invested dividends.

We present the estimation results of the present-value model in equations (4) to (6) using market-reinvested dividend data. The model is estimated by conditional maximum likelihood using data from 1946 to 2007 on the dividend growth rate and the price-dividend ratio. Panel A presents the estimates of the coefficients of the underlying processes (bootstrapped standard errors in parentheses). Panel B reports the resulting coefficients of the present-value model, which is given by ($pd_t = A - B_1(\mu_t - \delta_0) + B_2(g_t - \gamma_0)$). These parameters are non-linear transformations of the original present-value parameters. When interpreting the standard errors, it should be taken into account that the distribution of the coefficients is not symmetric. In Panel C we report the R^2 values for returns and dividend growth rates.

Panel A: Maximum likelihood estimates					
	Estimate	S.e.		Estimate	S.e.
δ_0	0.085	(0.019)	γ_0	0.059	(0.012)
δ_1	0.933	(0.148)	γ_1	-0.324	(0.282)
σ_μ	0.015	(0.014)	σ_g	0.094	(0.026)
$\rho_{d\mu}$	-0.422	(0.276)	σ_d	0.065	(0.022)
$\rho_{\mu g}$	0.905	(0.076)			
Panel B: Implied present-value model parameters					
A	3.596	(0.349)	ρ	0.968	
B_1	10.263	(3.439)	B_2	0.761	(2.883)
Panel C: R^2 values					
R^2_{Ret}	8.6%		R^2_{Div}	18.7%	

peaks, of which one is positive; the other is negative. Panels B and C show plots of the R-squared values for returns and dividend growth rates as a function of γ_1. The R^2 value for dividend growth rates also exhibits a bimodel shape, and perhaps surprisingly, the R^2 value is higher for the positive root than for the negative root of γ_1. Furthermore, the R^2 value for returns is also higher for the positive root of γ_1. The figures therefore illustrate that maximizing R^2 values is not necessarily equivalent to maximizing the likelihood (see also Harvey (1989)).

[Table IA.III about here]

[Figure IA.1 about here]

Table IA.III. Estimating a model with an AR(1)-process for expected growth rates in case of market-invested dividends.

In the column "> 0" we report the maximum likelihood estimates of equations (4) to (6), but using dividends that are reinvested in the market. In the first column, we impose the condition that the persistence coefficient of expected dividend growth rates is positive. We then define a grid for γ_1 between -0.9 and 0.8 with increments of 0.1, and compute for each of these values of γ_1 the likelihood while optimizing over all the other parameters.

		γ_1																	
	> 0	-0.9	-0.8	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	-0.1	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
δ_0	0.084	0.083	0.083	0.085	0.083	0.085	0.085	0.085	0.086	0.087	0.086	0.087	0.085	0.086	0.085	0.083	0.084	0.083	0.077
δ_1	0.933	0.957	0.954	0.952	0.949	0.9441	0.936	0.931	0.926	0.921	0.918	0.920	0.922	0.924	0.926	0.933	0.944	0.957	0.976
γ_0	0.058	0.059	0.059	0.060	0.059	0.060	0.059	0.059	0.059	0.059	0.058	0.059	0.058	0.058	0.058	0.057	0.058	0.059	0.059
γ_1	0.472	-0.900	-0.800	-0.700	-0.600	-0.500	-0.400	-0.300	-0.200	-0.100	0.000	0.100	0.200	0.300	0.400	0.500	0.600	0.700	0.800
σ_μ	0.022	0.010	0.010	0.011	0.011	0.012	0.014	0.015	0.017	0.019	0.020	0.021	0.021	0.022	0.023	0.022	0.022	0.020	0.017
σ_g	0.053	0.029	0.047	0.062	0.076	0.084	0.089	0.095	0.101	0.105	0.051	0.051	0.053	0.053	0.053	0.052	0.050	0.045	0.040
σ_d	0.107	0.109	0.101	0.092	0.082	0.075	0.069	0.062	0.057	0.053	0.109	0.109	0.108	0.106	0.106	0.106	0.107	0.108	0.109
$\rho_{g\mu}$	0.978	0.857	0.887	0.915	0.944	0.928	0.915	0.902	0.897	0.897	0.953	0.968	0.971	0.974	0.977	0.980	0.982	0.986	0.990
$\rho_{\mu d}$	0.208	0.093	-0.030	-0.158	-0.314	-0.369	-0.403	-0.431	-0.442	-0.442	0.255	0.249	0.237	0.225	0.213	0.200	0.187	0.169	0.142
R^2_{Ret}	0.105	0.075	0.077	0.078	0.079	0.081	0.084	0.086	0.088	0.090	0.092	0.094	0.097	0.100	0.103	0.106	0.106	0.104	0.095
R^2_{Div}	0.242	0.036	0.075	0.105	0.130	0.155	0.176	0.191	0.200	0.200	0.191	0.204	0.217	0.229	0.237	0.240	0.236	0.221	0.193
Log L	6.617	6.466	6.521	6.568	6.609	6.640	6.657	6.660	6.648	6.621	6.578	6.590	6.600	6.609	6.616	6.617	6.611	6.590	6.547

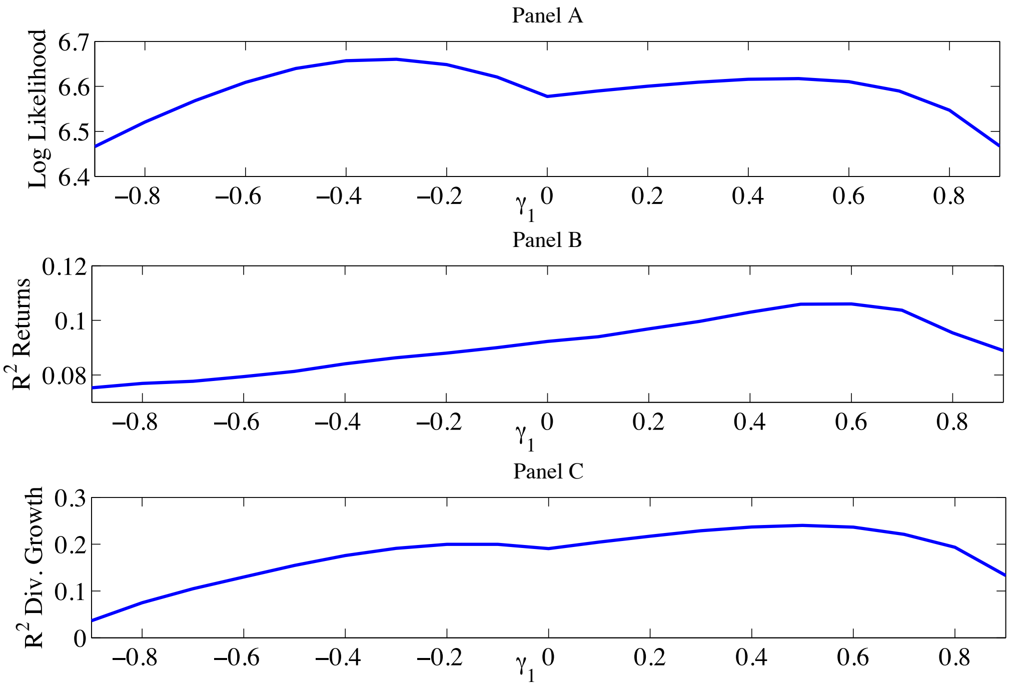

Figure IA.1. Log likelihood and R^2 values as a function of γ_1. The graph plots the log likelihood and the R^2 values as a function of the persistence of expected dividend growth, γ_1, using the system described in equations (7) to (9) and data where dividends are reinvested in the aggregate market. We define a grid for γ_1 between -0.9 and 0.9 with step size 0.1, and compute for each of these grid points the likelihood and R^2 values of the model while optimizing over all the other parameters.

IA.D. Likelihood Ratio Statistics

Table IA.IV. Likelihood-ratio tests.

We report the LR statistics for the tests described in Section IV; in particular, we report the results for the first four tests for the two specifications that we explore in this paper. “Cash” refers to the system in equations (4)to(6) using the data where dividends are reinvested at the risk-free rate. “Market” refers to the system in equations (7)to(9) using the data where dividends are reinvested in the aggregate stock market. Two asterisks (**) denotes that we reject the hypothesis at the 5% level and three asterisks (***) indicates that we reject the hypothesis at the 1% level. The critical values for the χ^2 statistic at the five percent level are given by 3.841, 5.991, 7.815, 9.488, and 11.070 for degrees of freedom equal to 1,2...,5, respectively. These five critical values are equal to 6.635, 9.210, 11.345, 13.277, and 15.086 for the 1% level.

					Parameters under H_0										
	LR	Sign	Log Lik. H_0	Log Lik. H_a	δ_0	δ_1	γ_0	γ_1	σ_μ	σ_g	σ_d	$\rho_{g\mu}$	$\rho_{\mu d}$	σ_M	ρ_M
Test for lack of return predictability															
Cash-reinv. dividends	**28.67**	***	7.0593	7.5218	0.0936	0	0.0637	0.9900	0	0.0065	0.0659	0	0	-	-
Market-reinv. dividends	**22.37**	***	6.4773	6.8381	0.0926	0	0.0666	0.9936	0	0.0057	0.0780	0	0	0.0607	0.8521
Test for lack of div. growth predictability															
Cash-reinv. dividends	**9.23**	**	7.3730	7.5218	0.0882	0.9261	0.0607	0	0.0164	0	0.0617	0	0.3494	-	-
Market-reinv. dividends	**29.59**	***	6.3609	6.8381	0.0833	0.9514	0.0587	0	0.0104	0	0.1222	0	0.2973	0	0
Test for lack of persistence in expected div. growth															
Cash-reinv. dividends	**8.26**	***	7.3886	7.5218	0.0882	0.9288	0.0610	0	0.0156	0.0605	0.0121	0.2550	0.2636	-	-
Market-reinv. dividends	**5.89**	**	6.7431	6.8381	0.0852	0.9262	0.0584	0	0.0174	0.0619	0.0470	0.7449	-0.2207	0.0501	0.6792
Test whether g_t and μ_t are equally persistent															
Cash-reinv. dividends	**8.60**	***	7.3831	7.5218	0.0867	0.9437	0.0595	0.9437	0.0157	0.0022	0.0617	0.9493	0.3090	-	-
Market-reinv. dividends	**5.10**	**	6.7558	6.8381	0.0782	0.9478	0.0548	0.9478	0.0166	0.0033	0.0764	0.9351	0.3541	0.0631	0.9254
Test for exclusion of ε_M															
Market-reinv. dividends	**11.00**	***	6.6607	6.8381	0.0854	0.9324	0.0591	-0.3253	0.0149	0.0939	0.0635	0.9064	-0.4212	0	0
Test $\rho_M = 0$															
Market-reinv. dividends	**6.93**	***	6.7264	6.8381	0.0853	0.9321	0.0584	0.4419	0.0209	0.0595	0.0633	0.9945	-0.1048	0.0479	0

IA.E. Long Sample Results: 1927-2007

So far, we have focused on the post-war sample, which is in line with a large share of the existing literature. In this appendix, we consider the sample period 1927-2007. The maximum likelihood estimates and R^2 values are presented in Table IA.V. Even though the results for returns are weaker, with an R^2 value below 2%, the results for dividend growth rates are stronger with an R^2 value of 45%.

Table IA.V. Estimation results of the model in (4)-(6) for the sample period 1927-2007.

We present the estimation results of the present-value model in equations (4) to (6) using cash-reinvested dividend data. The model is estimated by conditional maximum likelihood using data from 1927 to 2007 on the dividend growth rate and the price-dividend ratio. Panel A presents the estimates of the coefficients of the underlying processes. In Panel B we report the R^2 values for returns and dividend growth rates.

Panel A: Maximum likelihood estimates			
	Estimate		Estimate
δ_0	0.073	γ_0	0.045
δ_1	0.943	γ_1	0.261
σ_μ	0.014	σ_g	0.116
$\rho_{d\mu}$	-0.756	σ_d	0.004
$\rho_{\mu g}$	0.227		
Panel B: R^2 values			
R^2_{Ret}	1.9%	R^2_{Div}	45.0%