Toward Quantitative Literacy: Interesting Problems in Finance

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http://www.delta.edu/jaham

Fill in the blanks.

1. _______________ % of workers age 55 and up have saved less than $50,000 for retirement (not including the value of a primary residence).

2. The average household has a net worth of just $ _______________ at retirement, not including home equity.

3. The savings rate for all of 2006 was _______________ %.

4. ___________ % of all retirement plan participants who change jobs fail to roll over their accounts.

5. People in the 18 to 24 age bracket spend nearly _______________ % of their monthly income just on debt repayment.

6. Only about 1 in _______________ American households owes $8,000 or more on credit cards.

7. _______________ % of Americans owe nothing on their credit cards.

8. More than _______________ % of all U.S. households were in some phase of the foreclosure process last year.

9. More than _______________ % of undergraduates began the 2004 school year with credit cards.

10. _______________ % of undergraduates use their credit cards for tuition. The average student credit card balance is $ _______________

11. In 2005, students in a survey believed when they got older that they would earn an average salary of $ ____________ In 2005, adults with a bachelor's degree earned an average of $ ____________

12. _____________ % of teens and young adults say their parents taught them how to manage money.

13. _______________ % of high school graduates nationally have taken a course covering personal finance content.

14. The average 30-year fixed mortgage rate is approximately _______________

15. Subprime mortgages charge an interest rate lower than the prime lending rate in the first couple of years, but the rate goes up rather dramatically after a period of two to three years. True or False.

16. Each time you open a store credit card, _______________ points are taken off of your credit score.

17. In the first quarter of 2008, home values decreased by _______________ %.

APPS Finance TVM Solver

While cursor is blinking on the value to be calculated, enter ALPHA ENTER (SOLVE).

TVM Solver	
N=	**N** = number of payment periods
I%=	**I%** = annual interest rate (do not convert to a decimal; if APR = 9%, the I% = 9)
PV=	**PV** = present value (amount of the loan) or beginning lump sum investment
PMT=	**PMT** = per period payment amount
FV=	**FV** = future value
P/Y=	**P/Y** = number of payments per year
C/Y=	**C/Y** = number of compounding periods per year
PMT: END BEGIN	**PMT: END BEGIN** (When the regular payments are made: at the BEGINing of the period or at the END)

1. **Lump Sum Investment**: When Bud Uronner was born, his grandfather made an initial deposit of \$3,000 into an account for his college education. Assuming an interest rate of 6% compounded quarterly, how much will the account be worth in 18 years?

N= I%= PV= PMT= **FV=** P/Y= C/Y= PMT: END BEGIN	$A = P\left(1+\frac{r}{m}\right)^{mt}$ $A = 3000\left(1+\frac{.06}{4}\right)^{4(18)}$	**Explorations**: (a) Compare the effect of increasing **m** on the future value. Let **m** take on all the usual values: 1, 2, 4, 12, 52, 365. Complete the table below. Does a larger value of **m** increase the future value dramatically? Explain. (b) Compare the effect of increasing **r** on the future value. Let **r** take on all the values: 1%, 5%, 8%, 9%, 13%, 20%. Complete the table below. Does a larger value of **r** increase the future value dramatically? Explain.

m	A (r = .06; P = 3000; t = 18)
1	
2	
4	
12	
52	
365	

r	A (m = 4; P = 3000; t = 18)
.01	
.05	
.08	
.09	
.13	
.20	

2. **Rule of 72**: Orson Buggy wants his $5,000 investment to double in 6 years. What annual interest rate must he earn? Assume interest is compounded annually.

N= **I%=** PV= PMT= FV= P/Y= C/Y= PMT: END BEGIN	$A = P\left(1+\frac{r}{m}\right)^{mt}$ $10000 = 5000\left(1+\frac{r}{1}\right)^{1(6)}$	**Explorations**: • Compare the effect of changing **t** on the interest rate, **r**. Multiply **t** and **r** in each case. Let m = 1; A = 10000; P = 5000. Use the following values for **n** = **t**: 2, 3, 4, 6, 8, 9, 12, 18, 24, 36. Complete the table below. How is this exploration related to the rule of 72?

t	r	r * t
2		
3		
4		
6		
8		
9		
12		
18		
24		
36		

3. **Effective Annual Yield:**

(a) Find the effective rate corresponding to a nominal rate of 8.5% compounded quarterly.

►Eff(r%, m) =	$r_e = \left(1+\frac{r}{m}\right)^m - 1$ $r_e = \left(1+\frac{.085}{4}\right)^4 - 1$

(b) Find the nominal rate corresponding to an effective rate of 7.13%. Assume that the interest of the nominal rate is compounded daily.

►Nom(r%, m) =	$r_e = \left(1+\frac{r}{m}\right)^m - 1$ $.0713 = \left(1+\frac{r}{365}\right)^{365} - 1$

4. **Future Value Annuity**: How long will it take Dot Snice to accumulate $1,000,000 if she invests $3,000 per year at an annual interest rate of 8%? Assume interest is compounded annually.

N= I%= PV= PMT= FV= P/Y= C/Y= PMT: END BEGIN	$S = R\left[\frac{\left(1+\frac{r}{m}\right)^{mt}-1}{\frac{r}{m}}\right]$ $1000000 = 3000\left[\frac{\left(1+\frac{.08}{1}\right)^{1(t)}-1}{\frac{.08}{1}}\right]$	**Explorations**: • How long will it take to accumulate $1 million at 8% annual interest with different annual investments? • How long will it take a $3000 annual investment to grow to $1 million with different interest rates?

R	t (m = 1; r = .08; S = 1000000)
600	
1200	
3000	
7200	
12000	
15000	

r	t (m = 1; R = 3000; S = 1000000)
.01	
.05	
.08	
.09	
.13	
.20	

5. **Earn 32% Rate of Return the Easy Way**

(a) Many employers offer a 401K or 403B plan (tax sheltered annuity or TSA) that allows employees to invest for retirement. The beauty of the plan is that employees who invest $15,000 in a year, will pay federal taxes on $15,000 less in income – a tremendous tax savings. If we assume that the tax saved equals the rate of return on an investment, calculate the return on investment for the two employees below.

Salary	$50,000	$50,000
Investment in TSA	$15,000	$0
Taxable Income	$50,000 – $15,000 = $35,000	$50,000
Fed Tax Paid	$5,308	$9,058
State Tax Paid (4%)	(.04)($35000) = $1,400	(.04)($50000) = $2,000
Tax Savings:	($9,058 + $2,000) – ($5,308 + $1,400) = $4,350	
Rate of return:	$4,350/$15,000 = 29%	

(b) Repeat the above calculations to determine the tax savings of a second employee.

Salary	$80,000	$80,000
Investment in TSA	$15,000	$0
Taxable Income		
Fed Tax Paid	$12,902	$17,102
State Tax Paid (4%)		
Tax Savings:		
Rate of return:		

6. **Invest Early and Often**

(a) Much has been written about the importance of investing early and often. Two friends saved for retirement over a 40-year period in two different ways. Johnny on the Spot invested $4,000 per year at 8% annual interest for the first twenty years, then invested nothing over the last 20 years. During the last 20 years, his investments accumulated interest at 9% annual interest. Johnny Come Lately invested nothing for the first twenty years, but then invested $10,000 per year over the next 20 years at 9% annual interest.

- How much did Johnny on the Spot invest over the 40-year period?
- How much did Johnny Come Lately invest over the 40-year period?
- How much did Johnny on the Spot accumulate over the 40-year period?
- How much did Johnny Come Lately accumulate over the 40-year period?
- Who was the wiser investor and why?

(b) Compare the future values of the two Johnnys in the scenario below. Johnny on the Spot begins investing at age 25 and invests $3,000 for 30 years. From age 55 to age 65, he invests no additional funds. Johnny Come Lately begins investing at age 30 and invests $3,000 for 30 years. From age 60 to age 65, he invests no additional funds. What are the implications of waiting just 5 years to begin investing? Assume an annual interest rate of 8% throughout the 40-year period.

	Johnny on the Spot	Johnny Come Lately
Age at first Investment		
Number of years investing		
Annual Investment		
APR		
Total Amount Invested		
Future Value at Age 65		

(c) Compare the future values of the two Johnnys in the scenario below. Johnny on the Spot begins investing at age 20 and invests $3000 for 45 years. Johnny Come Lately begins investing at age 40 and invests $10,000 for 25 years. What are the implications of delayed investing? Assume an annual interest rate of 8% throughout the 40-year period.

	Johnny on the Spot	Johnny Come Lately
Age at first Investment		
Number of years until age 65		
Annual Investment		
APR		
Total Amount Invested		
Future Value at Age 65		

(d) Compare the future values of the two Johnnys in the scenario below. Johnny on the Spot begins investing at age 20 and invests $3,000 for 25 years. From age 45 to age 65, he invests no additional funds. Johnny Come Lately begins investing at age 40 and invests $3,000 for 25 years. What are the implications of delayed investing? Assume an annual interest rate of 8% throughout the 40-year period.

	Johnny on the Spot	Johnny Come Lately
Age at first Investment		
Number of years until age 65 (invests only first 25 years)		
Annual Investment		
APR		
Total Amount Invested		
Future Value at Age 65		

(e) In the first scenario above where Johnny on the Spot accumulates $1,025,875.40, how much would Johnny Come Lately have to invest each year during the 20-year period to accumulate the same amount as Johnny on the Spot after 40 years? What would be his total investment over the 20-year period?

7. **Present Value Annuity – Monthly Payment**: Megan Model borrows $25,000 at 7.53% compounded monthly. If she wishes to pay off the loan after 15 years, how much would the monthly payment be?

N= I%= PV= **PMT=** FV= P/Y= C/Y= PMT: END BEGIN	$P = R\left[\frac{1-\left(1+\frac{r}{m}\right)^{-mt}}{\frac{r}{m}}\right]$ $25000 = R\left[\frac{1-\left(1+\frac{.0753}{12}\right)^{-12(15)}}{\frac{.0753}{12}}\right]$	**Explorations**: (a) Complete the left table below. Compare the effect of decreasing **r** on the monthly payment. By how much does the monthly payment decrease if **r** goes down by 1%? (b) Complete the right table below. Compare the effect of decreasing **t** on the monthly payment.

r	R (m = 12; t = 15; S = 25000)
9%	
8%	
7%	
6%	
5%	
4%	

t	R (m = 12; r = .065; S = 135000)
10	
15	
16	
20	
25	
30	

8. **Managing Debt: Cost of Home Ownership**: Bob and Barb Noxious took out an $182,300 loan at 8.5% interest for 30 years for the purchase of a new house. The loan requires monthly mortgage payments.

(a) What is the monthly payment for this mortgage?

N=
I%=
PV=
PMT=
FV=
P/Y=
C/Y=
PMT: END BEGIN

(b) If you paid each of the 360 payments over the 30-year period, how much interest did you pay for the $182,300 house over the life of the loan? What was your total payout over the 30-year period?

Total Payout = (Monthly Payment) × (Number of payments)
Principal Paid = (Original Loan Amount) – (Present Value of Loan); At the end of 30 years, PV = 0.
Interest Paid = (Total Payout) – (Principal Paid)

∑Int(first pmt #, last pmt #)= Total Home Cost =	**Explorations**: • What monthly payment would lead to a $1 million payout over 30 years? What original home value would lead to a total payout of $1 million? Assume an annual percentage rate of 8.5%.

(c) If you wanted to pay off the loan after having paid 10 years of payments, how much would you have to pay? N = number of payments remaining, not the number of payments already made.

N=
I%=
PV=
PMT=
FV=
P/Y=
C/Y=
PMT: END BEGIN

(d) How much interest would have been paid over the 10 years? How much equity would they have in the house at this time? Assume the value of the house has grown 2% per year up to a value of $222222.68?

Total Payout = (Monthly Payment) × (Number of payments)
Principal Paid = (Original Loan Amount) – (Present Value of Loan)
Interest Paid = (Total Payout) – (Principal Paid)
Equity = (Net Home Value) – (Present Value of Loan)

∑Int(first pmt #, last pmt #) =	∑Prn(first pmt #, last pmt #) =
(Be sure to enter the original TVM Solver inputs from Part (a) before using these functions.	

(e) Calculate the present value of the loan for different values of **t** (and hence, **n**). How does the present value change as t increases? N = number of payments remaining, not the number of payments already made.

t = n	PV (Original Loan = $182300; m = 12; r = .085; PMT = 1401.73)
1	
2	
3	
5	
10	
20	
25	
28	
29	
30	

(f) Suppose Bob and Barb bought their home 10 years ago and made monthly payments as scheduled. They plan to move in two years. They could refinance for 7.25% right now on a new 20-year mortgage, but closing costs would be $1800. Should they refinance? Assume that they will roll over the closing costs into the new mortgage.

Calculate Original Monthly Payment	Calculate the Present Value	Calculate the New Monthly Payment
N= I%= PV= 182300 **PMT=** FV= P/Y= C/Y= PMT: END BEGIN	N= I%= PV= **PMT=** FV= P/Y= C/Y= PMT: END BEGIN	N= I%= PV= **PMT=** FV= P/Y= C/Y= PMT: END BEGIN

	Current Mortgage (8.5%)	New Mortgage (7.25%)
Present Value		
Monthly Payment?		
Savings per month?		
Number of months to recoup the closing costs?		

(g) What if the refinance rate was 7.75%? Would the refinance still make sense for Bob & Barb?

	Current Mortgage (8.5%)	New Mortgage (7.75%)
Present Value		
Monthly Payment?		
Savings per month?		
Number of months to recoup the closing costs?		

(h) If, on the original loan, they paid an additional $100 per month, how long would it take to pay off the loan?

N= I%= PV= PMT= FV= P/Y= C/Y= PMT: END BEGIN	**Explorations**: • By how many years would the loan term be reduced if an additional $200 was added to each payment? an additional $300? • What additional payment would be required to reduce the term of the loan to 15 years? • When establishing a mortgage, which is usually lower, a 15-year fixed mortgage rate or a 30-year fixed mortgage rate?

Additional $100: t = ____________; Years reduced = ________________

Additional $200: t = ____________; Years reduced = ________________

Additional $300: t = ____________; Years reduced = ________________

If term = 15 years, Additional Payment = __________________

(i) Ask a friend or relative about their current mortgage. Write the present value, monthly payment, and interest rate of the current mortgage in the table below. Complete the rest of the table. Write a couple of sentences describing the advice you would give your friend or relative.

	Current Mortgage (r = ___%)	New Mortgage (r = 6%)
Present Value		
Monthly Payment?		
Savings per month?		
Number of months to recoup the closing costs? Assume closing costs are $2000.		

9. **Managing Debt: Purchasing a Car**.

(a) Suppose that you are going to finance the purchase of a new $21,000 car. There are three financing options available to you: 1.9% financing for 3 years, 3.9% financing for 4 years, or 5.9% financing for 5 years. Compare the financing costs for each of the three loans. Which would be best for you and why?

1.9% financing for 3 years	3.9% financing for 4 years	5.9% financing for 5 years
N=	N=	N=
I%=	I%=	I%=
PV= 21000	PV= 21000	PV= 21000
PMT=	**PMT=**	**PMT=**
FV= 0	FV= 0	FV= 0
P/Y= 12	P/Y= 12	P/Y= 12
C/Y= 12	C/Y= 12	C/Y= 12
PMT: END BEGIN	PMT: END BEGIN	PMT: END BEGIN

$21,000 Car Loan			
Loan Term	3 Years (1.9%)	4 Years (3.9%)	5 Years (5.9%)
Monthly Payments			
Total Number of Payments	36	48	60
Total payout during the term			
Cost to Finance - Interest			

10. **Cash Back or Low Rate?**

(a) Paige is offered two options when purchasing a new $17,000 car. Option 1 offers 6.75% financing for 4 years and $2500 “cash back.” Option 2 offers 4.75% financing for 5 years with no cash back. The financing requires monthly payments. Find the monthly payment for each financing option. Assume that the cash back in Option 1 will be used to reduce the amount of the original loan. If Paige’s goal is to pay the minimum amount for financing over the life of the loan, which option should she choose? Explain why using specific numbers.

Option 1	Option 2
N=	N=
I%=	I%=
PV=	PV=
PMT=	PMT=
FV= 0	FV= 0
P/Y= 12	P/Y= 12
C/Y= 12	C/Y= 12
PMT: END BEGIN	PMT: END BEGIN

(b) In problem (a), assume Option 2 offers 4.5% financing. Now which option is best?

Option 1	Option 2
N=	N=
I%=	I%=
PV=	PV=
PMT=	PMT=
FV= 0	FV= 0
P/Y= 12	P/Y= 12
C/Y= 12	C/Y= 12
PMT: END BEGIN	PMT: END BEGIN

(c) In problem (a), how much cash back should be offered so that the total spent on the car at the end of the terms is equal for the two options?

11. **Managing Debt: Cost to Own (Leasing & Residuals)**

(a) Which vehicle should you purchase to minimize the "cost to own," a Dodge Caravan that costs $21011 and has a residual of 31.8% after 3 years, or a Toyota Sienna that costs $37695 and has a residual of 60% after 3 years? Assume an annual interest rate of 8%, and that you will sell the vehicle at its residual value after three years.

To determine the cost to own, a residual value is used. The residual value is essentially the proportion of the vehicle's original value that the vehicle will be worth in 3 years. (That is, a measure of depreciation.) This residual value is the number used to determine the monthly payment for a lease.

Dodge Caravan	**Toyota Sienna**
N= 36 I%= 8 PV= 21011 PMT= FV= 0 P/Y= 12 C/Y= 12 PMT: END BEGIN	N= 36 I%= 8 PV= 37695 PMT= FV= 0 P/Y= 12 C/Y= 12 PMT: END BEGIN

	Dodge Caravan	**Toyota Sienna**
List Price	$21,011	$37,695
8% 3-year loan pymt		
Total Payments		
Residual	31.8%	60%
Residual Value		
Total Cost		
Cost per Month		

(b) Repeat the above calculations to determine which car has the lowest cost to own, A Chevy Cavalier or a Toyota Camry. The Chevy Cavalier costs $21011 and has a residual of 26.3% after 3 years, and a Toyota Camry costs $29650 and has a residual of 63% after 3 years? Assume an annual interest rate of 8%, and that you will sell the vehicle at its residual value after three years.

	Chevy Cavalier	Toyota Camry
List Price	$17,510	$29,650
8% 3-year loan pymt		
Total Payments		
Residual	26.3%	63%
Residual Value		
Total Cost		
Cost per Month		

12. **Managing Debt: Paying Off a Credit Card**

Dell has advertised a Dimension E521 computer for $1149 ($1218 after tax) or $35 per month. You are in need of a new computer and this model seems to satisfy all of your needs. Suppose that you pay only the minimum due of $35 (at 19.99% APR) each month on your new computer.

(a) How long will it take you to pay off the computer? How much will you have paid on the $1218 balance when the computer is finally paid off?

N=
I%=
PV=
PMT=
FV=
P/Y=
C/Y=
PMT: END BEGIN

(b) Suppose your friend purchases the same computer and has the same beginning balance or $1218. Because your friend has bad credit, the annual interest rate is 29.99%. How long will it take your friend to pay off the computer? How much will your friend have paid on the $1218 balance when the computer is finally paid off?

N=
I%=
PV=
PMT=
FV=
P/Y=
C/Y=
PMT: END BEGIN

(c) How much would you need to invest in a sinking fund each month at 5% interest compounded monthly to accumulate the $1218 needed to purchase the computer in 2 years?

(d) How many months would it take to accumulate the $1218 needed to purchase the computer if you invest $35 in a sinking fund earning 5% interest?

13. **Subprime Mortgage Crisis**

Read the articles at
http://money.howstuffworks.com/personal-finance/real-estate/subprime-mortgage1.htm
http://money.howstuffworks.com/personal-finance/real-estate/subprime-mortgage2.htm

Excerpt: "One of the more common subprime loans has an adjustable-rate mortgage (ARM) attached. ARMs have become increasingly popular in recent years due to their initial low monthly payments and low interest rates. Introductory rates for ARMS typically last two or three years. The rate is then adjusted every six to 12 months and can increase by as much as 50 percent or more. If you hear about a 2/28 or a 3/27 ARM, the first number refers to the number of years at the introductory rate, the second to the remaining period of the loan with the fluctuating rate.

Interest-only options are also often attached to subprime ARMs. Let's look at a 2/28 interest-only ARM. This loan allows you to pay only on the interest during the two year introductory period at a lower set rate. After that, the full amount of the loan is recalculated over the remaining 28 years with a new rate. According to Bankrate.com, the difference in the monthly payments on a 2/28 interest-only subprime ARM can be dramatic.

As you can see, it's likely when the introductory period runs out that you'll be in store for a much higher monthly payment."

Loan = $200,000	Rate	Monthly payment	Monthly payment/Interest Only
First two years	7%	$1,330.60	Varies: $1166.67 down to $1143.20
Third year	11%	$1,822.48	$1922.96

(a) In a $131,250 subprime loan with a 3/27 ARM, the initial interest rate is 8% and it will remain 8% for three years; at the end of the first three years it will increase to 12%. Complete the table below. For both a regular and interest-only loan, by how much will the monthly payment increase after the third year when the rate increases?

	Regular 3/27 ARM	Interest-only 3/27 ARM
Monthly Payments (first 3 years)		Varies: $875.00 down to $851.94
Interest Paid over 1st 3 years		
Principal Paid over 1st 3 years		
Present Value of Loan after 3 years		
Monthly Payment after 3rd year		
Increase in payment		

(b) Suppose the home value depreciated 14.5% per year during the first three years of the loan. (This is a pretty close estimate of the actual drop in median home values in the Detroit area over the past three years.) Compare the value of the home to the outstanding balance of the loan. Assume that there was no down payment required to secure the loan. (Use the Regular 3/27 ARM numbers.) What are your options if faced with this scenario: to owe more on the loan than the home is worth? What would you do if faced with this scenario?

(c) In a $251,750 subprime loan with a 2/28 ARM, the initial interest rate is 7% and it will remain 11% for two years; at the end of the first two years it will increase to 11%. Complete the table below. For both a regular and interest-only loan, by how much will the monthly payment increase after the second year when the rate increases?

	Regular 2/28 ARM	Interest-only 2/28 ARM
Monthly Payments (first 2 years)		Varies: $1468.54 down to $1439.00
Interest Paid over 1st 2 years		
Principal Paid over 1st 2 years		
Present Value of Loan after 2 years		
Monthly Payment after 2nd year		
Increase in payment		

(d) Suppose the home value depreciated 3.5% per year during the first two years of the loan. Compare the value of the home to the outstanding balance of the loan. Assume that there was no down payment required to secure the loan. (Use the Regular 2/28 ARM numbers.) What are your options if faced with this scenario: to owe more on the loan than the home is worth? What would you do if faced with this scenario?

(e) Repeat part (b), but assume that the original value of the home was $164062.50 and the buyer was required to make a down payment of 20% or $32812.50 to secure the loan. Notice that the loan amount would still be $131250. Compare the value of the home to the outstanding balance of the loan. (Use the Regular 3/27 ARM numbers and the same depreciation rate.) What would you do if faced with this scenario?

(f) Repeat part (d), but assume that the original value of the home was $314687.50 and the buyer was required to make a down payment of 20% or $62937.50 to secure the loan. Notice that the loan amount would still be $251750. Compare the value of the home to the outstanding balance of the loan. (Use the Regular 2/28 ARM numbers and the same depreciation rate.) What would you do if faced with this scenario?