Properties of Logarithms

Tools for solving logarithmic and exponential equations

Let’s review some terms.

When we write
$\log_5 125$

5 is called the ***base***
125 is called the ***argument***

Logarithmic form of $5^2 = 25$ is

$$\log_5 25 = 2$$

For all the laws

a, M and N > 0

$a \neq 1$

r is any real

Remember ln and log

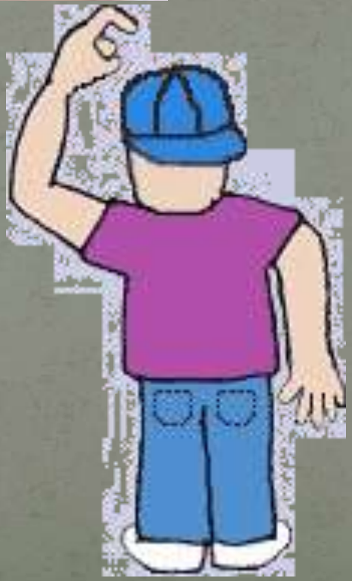
- ln is a short cut for $\log_e$
- log means $\log_{10}$

Easy ones first :

$$\log_a 1 = 0$$

since $a^0 = 1$

$\log_3 1 = ?$



$\log_3 1 = ?$

$\log_a 1 = 0$

$$\log_3 1 = 0$$

$$\log_a 1 = 0$$

$\ln 1 = ?$

$\ln 1 = ?$

$\log_a 1 = 0$

$$\ln 1 = 0$$

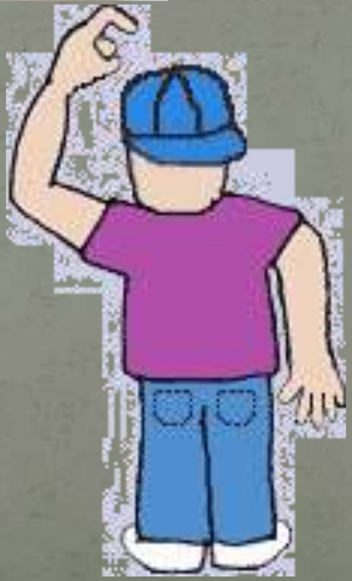
$$\log_a 1 = 0$$

Another easy one :

$$\log_a a = 1$$

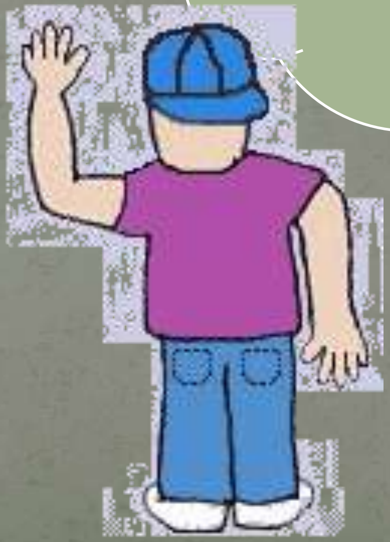
since $a^1 = a$

$\log_5 5 = ?$



$\log_5 5 = ?$

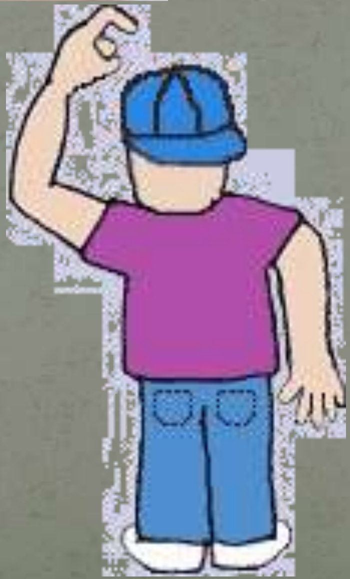
$\log_a a = 1$



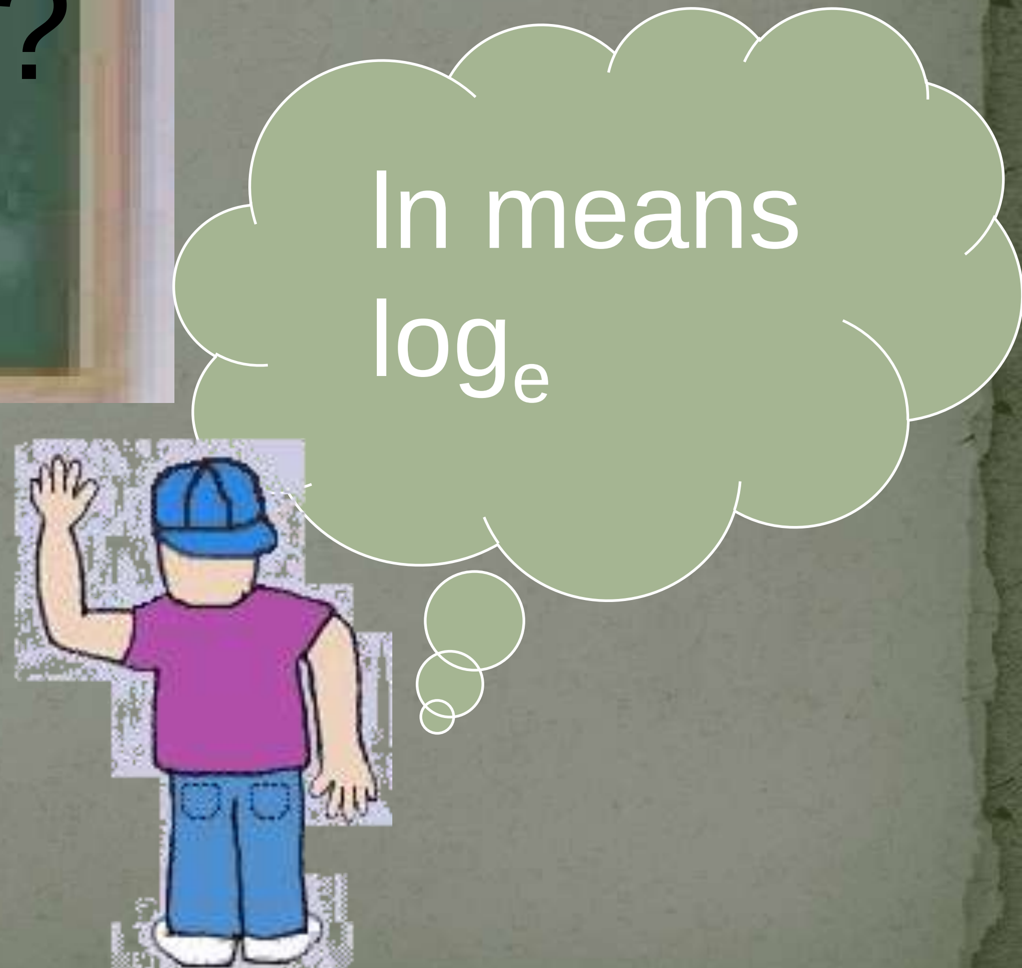
$\log_5 5 = 1$

$\log_a a = 1$

$\ln e = ?$



$$\ln e = \log_e e = ?$$



$$\ln e = \log_e e = ?$$

$$\log_a a = 1$$

$$\ln e = 1$$

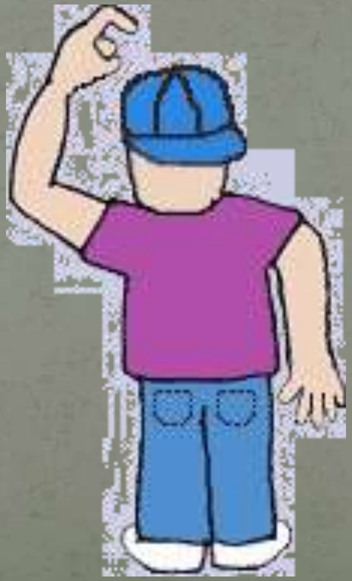
$$\log_a a = 1$$

Just a tiny bit harder :

$$\log_a a^r = r$$

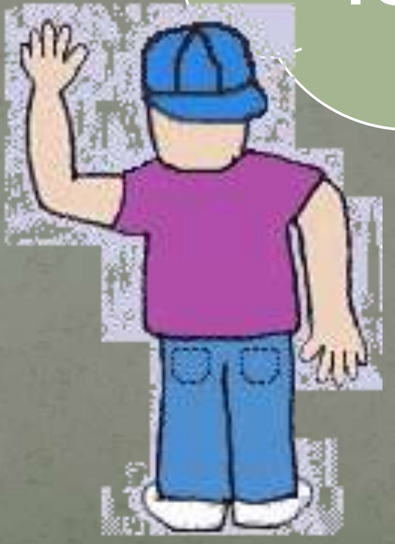
since $a^r = a^r$

$\ln e^{3x} = \ ?$

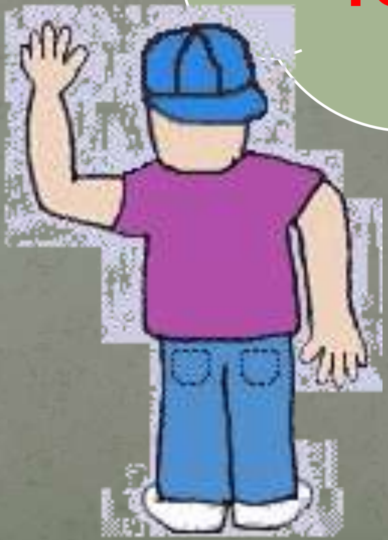


$$\ln e^{3x} = \log_e e^{3x} = ?$$

ln means $\log_e$



$\ln e^{3x} = \log_e e^{3x} = ?$

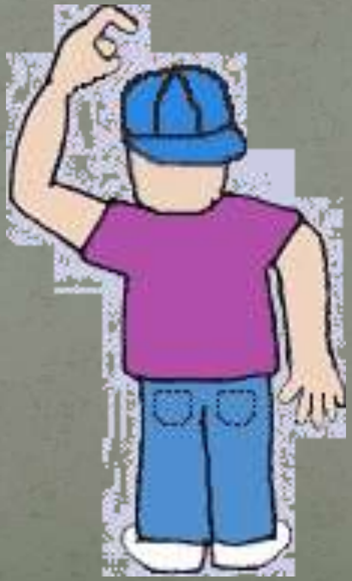


Use

$\log_a a^r = r$

$\ln e^{3x} = \log_e e^{3x} = 3x$

$\log(10^{5y}) = ?$



$\log(10^{5y}) = ?$

log means $\log_{10}$

$$\log(10^{5y}) = \log_{10} 10^{5y} = ?$$

$$\log_a a^r = r$$

$$\log(10^{5y}) = \log_{10} 10^{5y} = 5y$$

$\log_a(MN) = \log_a(M) + \log_a(N)$

Evidence that it works (not a proof):

$$\log_5(125) = 3$$

$$\log_5(125) = \log_5(25) + \log_5(5)$$

$$3 = 2 + 1$$

$\log_a(M/N) = \log_a(M) - \log_a(N)$

Evidence that it works (not a proof):

$\log_5(25) = 2$

(You know that 25 = 125 / 5)

$\log_5(25) = \log_5(125) - \log_5(5)$

$2 = 3 - 1$

$\log(2x) = ?$

$\log(2x) = ?$

$\log_a(MN) = \log_a(M) + \log_a(N)$

$$\log(2x) = \log(2) + \log(x)$$

$$\log_a(MN) = \log_a(M) + \log_a(N)$$

$$\ln \frac{2}{x-3}$$

$$\ln \frac{2}{x-3}$$

$\log_a(M/N) = \log_a(M) - \log_a(N)$

$$\ln\frac{2}{x-3} = \ln(2) - \ln(x-3)$$

$\log_a(M/N) = \log_a(M) - \log_a(N)$

Power Rule :

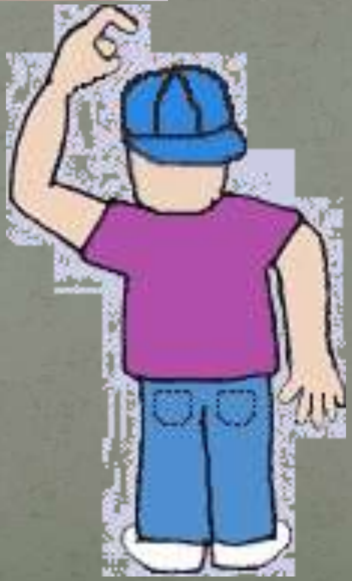
$$\log_a M^r = r \log_a M$$

Think of it as repeated uses of

$$\log_a(MM) = \log_a(M) + \log_a(M)$$
$$= 2 \log_a(M)$$

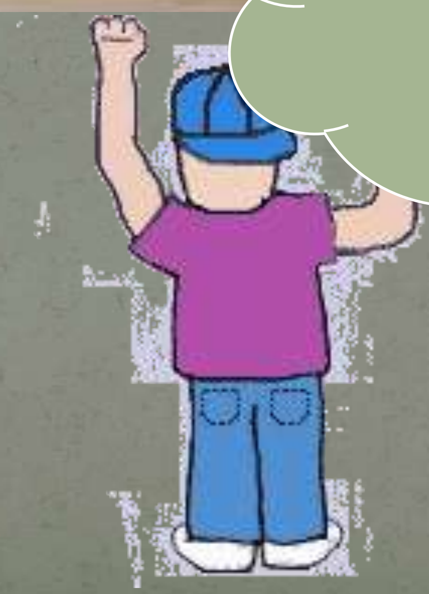
r times

$\ln(x^2)$



$\ln(x^2)=$

$\log_a(M^r) =$
$r \log_a M$



$$\ln(x^2) = 2 \ln x$$

$$\log_a(M^r) = r \log_a M$$

- $\ln(x^2y)$

- $\ln(x^2y)$

$\log_a(MN) = \log_a(M) + \log_a(N)$

- $\ln(x^2y) = \ln(x^2) + \ln(y)$

$\log_a(MN) =$
$\log_a(M) + \log_a(N)$

- $\ln(x^2y) = \ln(x^2) + \ln(y)$

  $= 2\ln(x) + \ln(y)$

$\log_a(M^r) = r\log_a M$

NEVER DO THIS

- $\log(x + y) = \log(x) + \log(y)$ (ERROR)
- WHY is that wrong?
  - Log laws tell use that $\log(x) + \log(y) = \log(xy)$
    Not $\log(x + y)$

This uses $\log_a(M/N) = \log_a(M) - \log_a(N)$

Consider 5 = 5

You know that the

and the are equal

So if you knew that :

$$\log_a M = \log_a N$$

you would know that

$$M = N$$

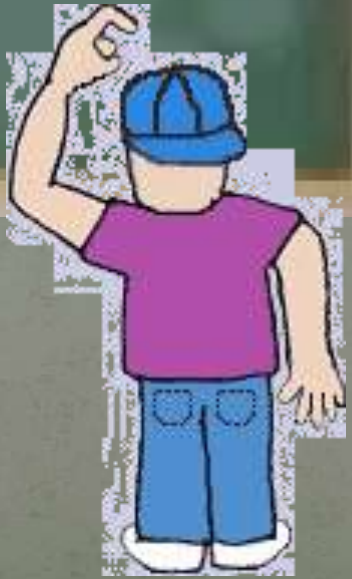
And vice versa, suppose

$$M = N$$

Then it follows that

$$\log_a M = \log_a N$$

$$\ln(x + 7) = \ln(10)$$



$$\ln(x + 7) = \ln(10)$$

$$x + 7 = 10$$

$$\ln(M) = \ln(N)$$

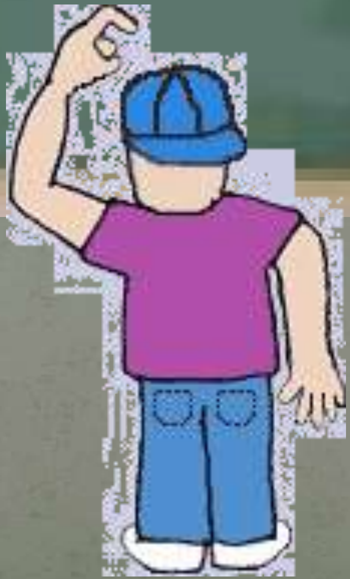
$$\ln(x + 7) = \ln(10)$$

$$x + 7 = 10$$

$$x = 3$$

subtract 7

$$\log_3(x + 5) = \log_3(2x - 4)$$



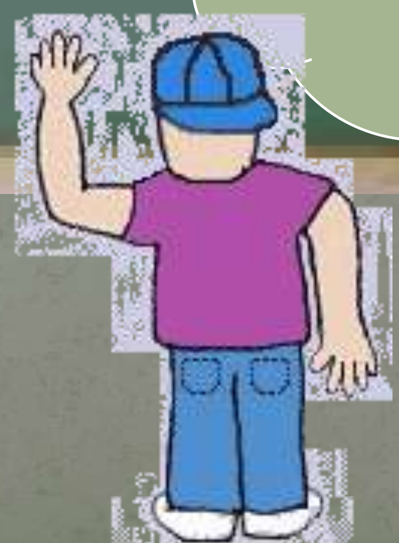
$$\log_3(x + 5) = \log_3(2x - 4)$$

$$\log(M) = \log(N)$$

$$\log_3(x + 5) = \log_3(2x - 4)$$

$$x + 5 = 2x - 4$$

$$\log(M) = \log(N)$$



$$\log_3(x + 5) = \log_3(2x - 4)$$

$$x+5 = 2x - 4$$

$$9 = x$$

$$3^{2x} = 5^x$$

$3^{2x} = 5^{x}$

If $M = N$

then

$\ln M = \ln N$

$$3^{2x} = 5^x$$

$$\ln(3^{2x}) = \ln(5^x)$$

If $M = N$

then

$\ln M = \ln N$

$$3^{2x} = 5^{x}$$

$$\ln(3^{2x}) = \ln(5^{x})$$

$\log_a(M^r) = r \log_a M$

$$3^{2x} = 5^{x}$$

$$\ln(3^{2x}) = \ln(5^{x})$$

$$2x \ln(3) = x \ln(5)$$

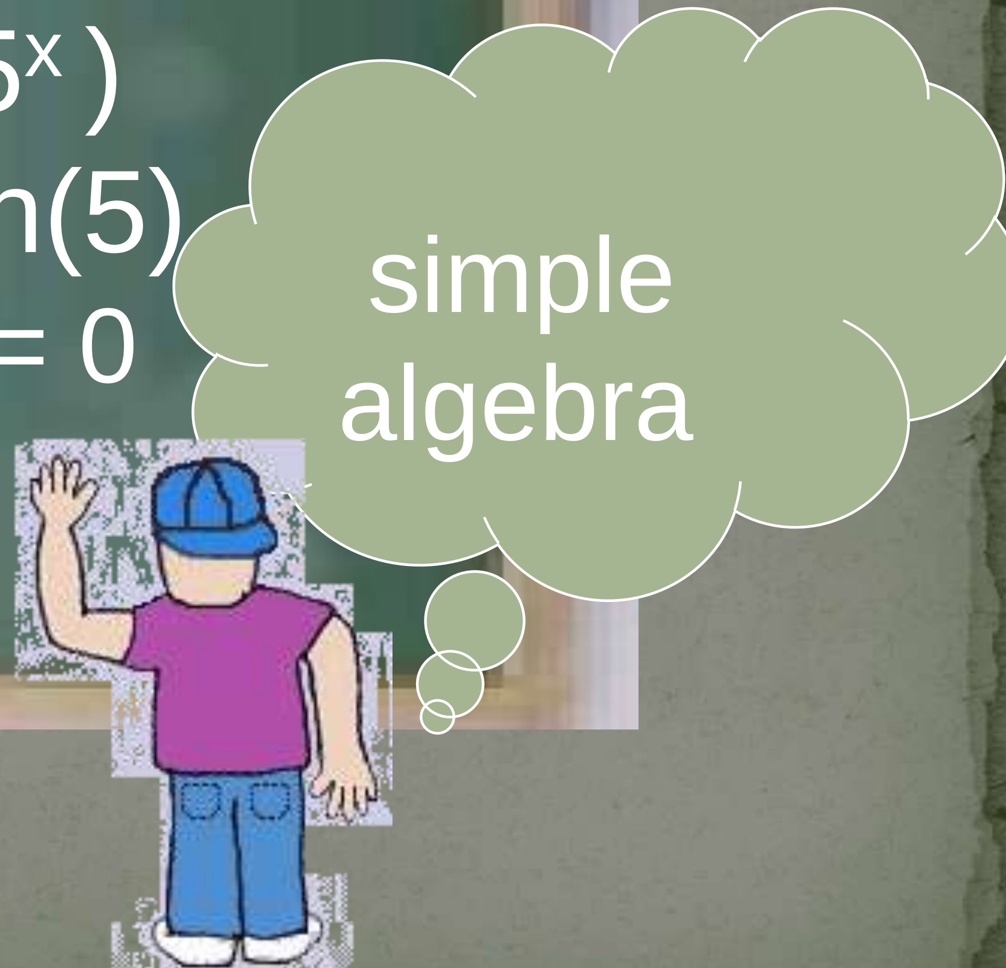
Get x terms on same side

$$3^{2x} = 5^{x}$$

$$\ln(3^{2x}) = \ln(5^{x})$$

$$2x \ln(3) = x \ln(5)$$

$$2x(\ln 3) - x \ln(5) = 0$$



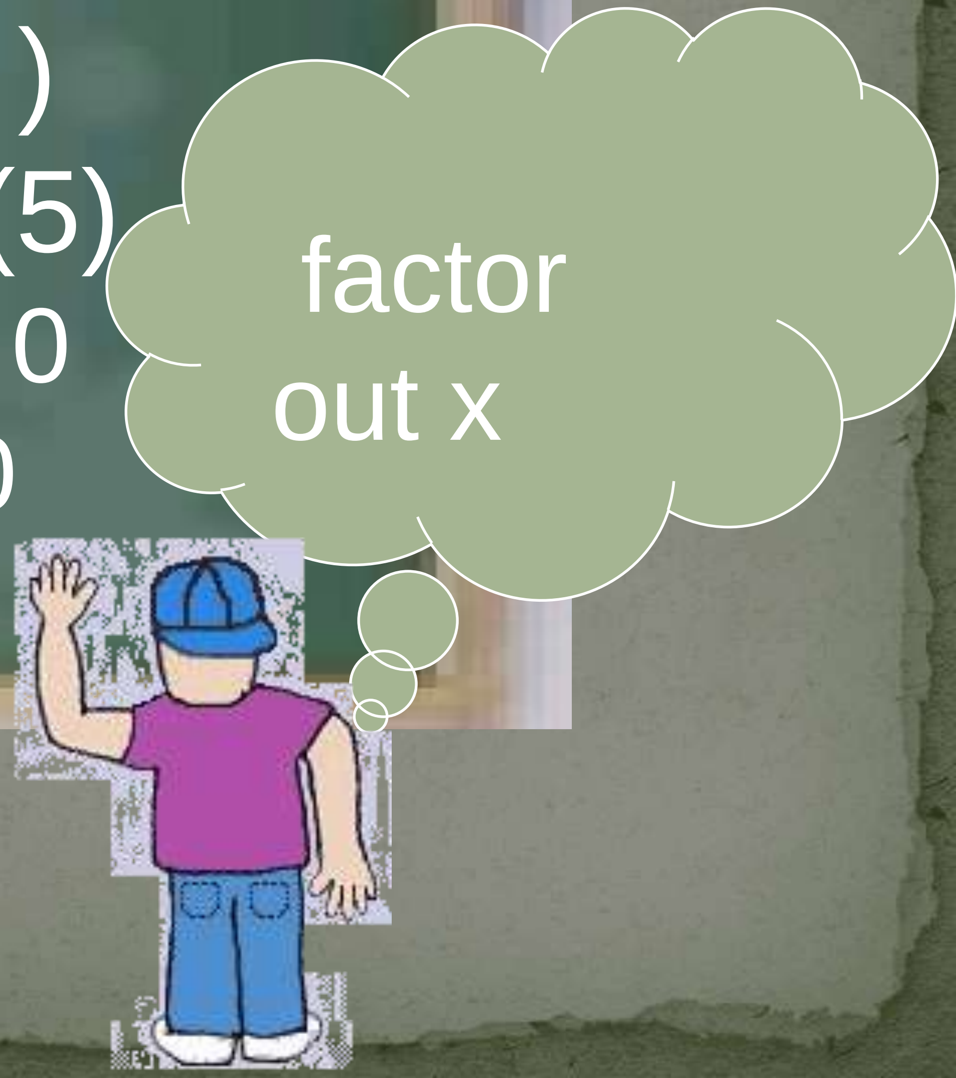
$$3^{2x} = 5^{x}$$

$$\ln(3^{2x}) = \ln(5^{x})$$

$$2x \ln(3) = x \ln(5)$$

$$2x(\ln 3) - x \ln(5) = 0$$

$$x[2\ln(3) - \ln(5)] = 0$$



$$3^{2x} = 5^x$$

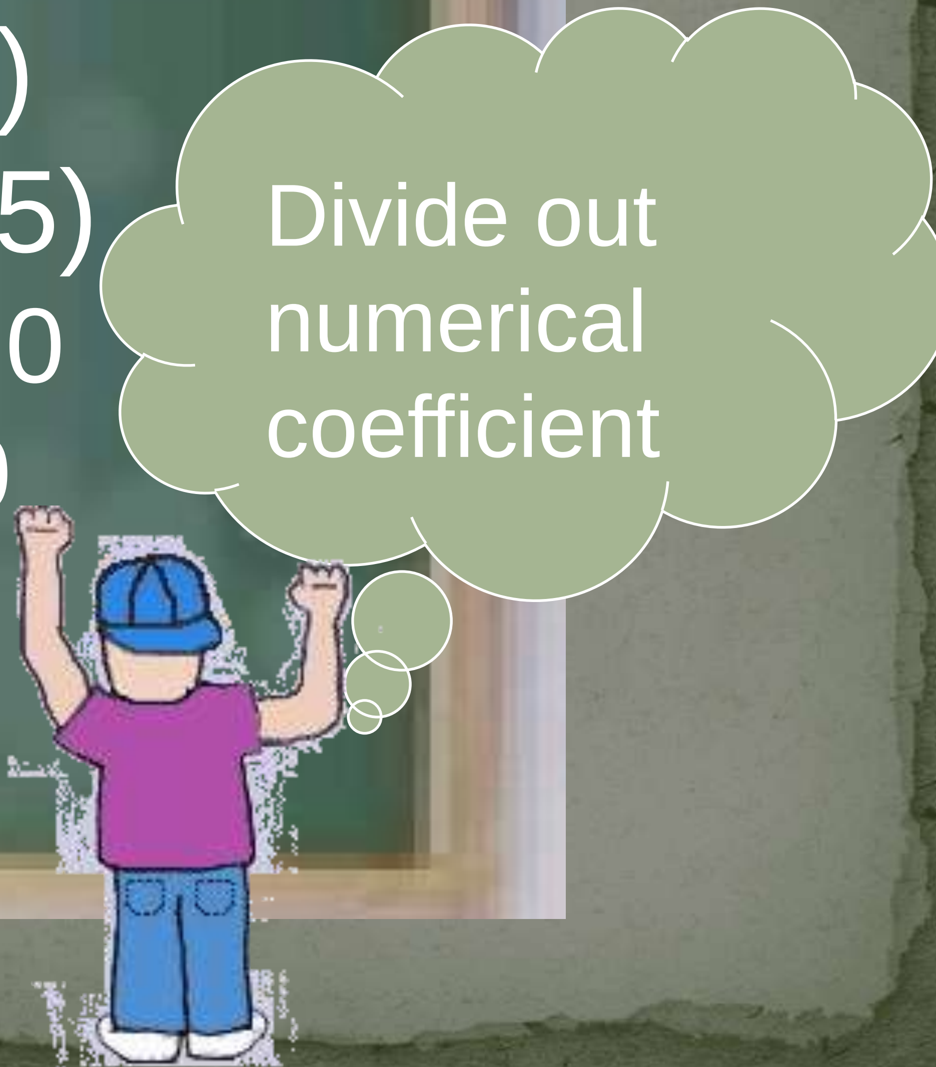
$$\ln(3^{2x}) = \ln(5^x)$$

$$2x \ln(3) = x \ln(5)$$

$$2x(\ln 3) - x \ln(5) = 0$$

$$x[2\ln(3) - \ln(5)] = 0$$

$$x = \frac{0}{2\ln(3) - \ln(5)}$$



$$3^{2x} = 5^{x}$$

$$\ln(3^{2x}) = \ln(5^{x})$$

$$2x \ln(3) = x \ln(5)$$

$$2x(\ln 3) - x \ln(5) = 0$$

$$x[2\ln(3) - \ln(5)] = 0$$

$$x = \frac{0}{2\ln(3) - \ln(5)} = 0$$

Simplify the fraction

Change of Base Formula : When you need to approximate $\log_5 3$

$$\log_a M = \frac{\ln M}{\ln a}$$

Change of Base Formula : When you need to approximate $\log_5 3$

$$\log_3 5 = \frac{\ln 5}{\ln 3}$$

Here's one not seen as much as some of the others:

$$a^{\log_a M} = M$$

$$a^{\log_a M} = M$$

Here's an example

$$e^{\log e 3x} = 3x$$

$$e^{\ln 3x} = 3x$$