Top Ten Things Every College Graduate Should Know and How This Can Be Achieved By Curve Fitting

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A student should be able to . . .

1. Research, Analyze, and Report
2. Analyze Data by Inspection
3. Verify
4. Models Get Modified
5. Use Models to Support an Argument
6. Ambiguous Authentic Situations
7. Correct terminology
8. Convert Units
9. Sift Through Large Amounts of Information
10. Mathematics is Truly Useful

Student Quote

Explorations are helpful and introduce new ideas in an easy manner. I like them—easy to follow and they're short. The class is not hard if you do the homework, projects, and pass all the tests.

Student Quote

Like any other art form, math takes practice and a lot of attention. I guess more like a girlfriend.

Every college graduate should . . .

be able to perform research, analyze the information, and report conclusions.

Modeling an Authentic Situation

Percentages of Americans Who Are Baseball Fans

Year	Percent
1999	54
2001	
2003	
2005	
2007	
2008	

Source: *The Gallup Organization*

Modeling an Authentic Situation

Percentages of Americans Who Are Baseball Fans

Year	Percent
1999	54
2001	51
2003	50
2005	48
2007	44
2008	43

Source: *The Gallup Organization*

Every college graduate should . . .

be proficient in analyzing data by inspecting a table or graph.

Analyze Data by Inspecting a Table or Graph

Percentages of Americans Who Are Baseball Fans

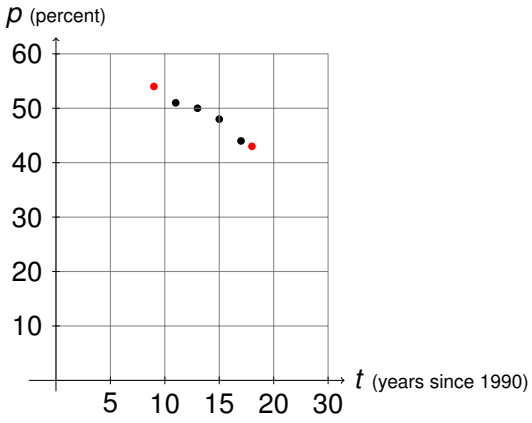
Year	Percent
1999	54
2001	51
2003	50
2005	48
2007	44
2008	43

Source: *The Gallup Organization*

By inspecting the table,

- determine the type of relationship.
- make a prediction.

Scattergram of Baseball Data



Finding a Linear Model

p: percent
$t = 0$: 1990
Use the "good" points $(9, 54)$ and $(18, 43)$:

$$m = \frac{43 - 18}{18 - 9} = -1.22$$

$$p = -1.22t + b$$
$$54 = -1.22(9) + b$$
$$64.98 = b$$

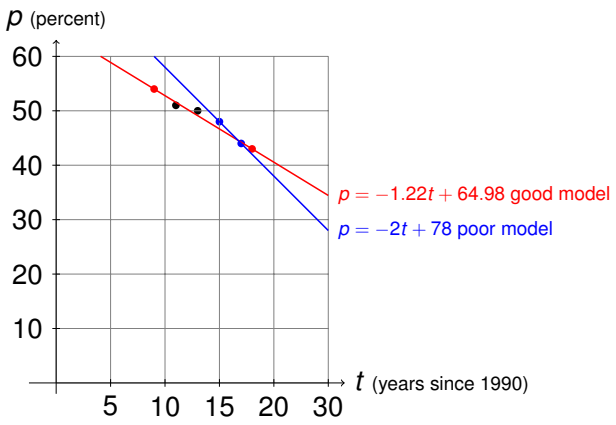
$$p = -1.22t + 64.98$$

Every college graduate should . . .

know the value of verifying results.

Verify the Fit

Selecting "Good" Points



Meaning of Parameters

$p = -1.22t + 64.98$

Meaning of Slope -1.22
Percentage is decreasing by 1.22 percentage points per year.

Meaning of constant 64.98
The percentage was 64.98% in 1990.

Evaluating a Function

$p = f(t) = -1.22t + 64.98$

Predict the percentage in 2010.
$f(20) = -1.22(20) + 64.98 = 40.58$
40.58%

Solving an Equation and Evaluating an Inverse Function

$p = f(t) = -1.22t + 64.98$

Predict when the percentage will be 33.33%.

$$\begin{aligned} 33.33 &= -1.22t + 64.98 \\ -31.65 &= -1.22t \\ t &\approx 26 \ (2016) \end{aligned}$$

$$f^{-1}(p) = \frac{p - 64.98}{-1.22}$$

$$f^{-1}(33.33) = \frac{33.33 - 64.98}{-1.22} \approx 26(2016)$$

Model Breakdown

Find the t-intercept of the model. What does it mean in this situation?

$$\begin{aligned} 0 &= -1.22t + 64.98 \\ 1.22t &= 64.98 \\ t &\approx 53 \end{aligned}$$

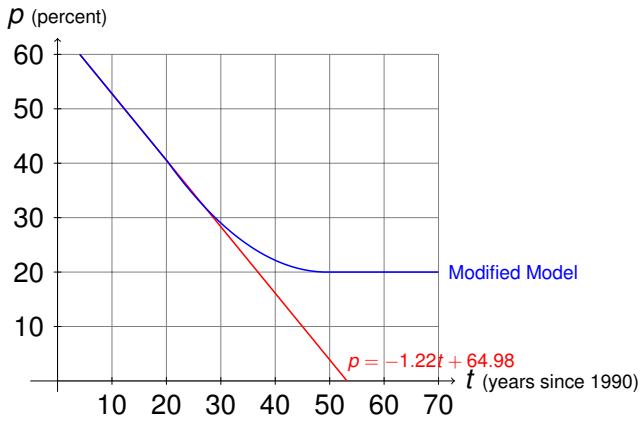
t-intercept: $(53, 0)$

No Americans will be baseball fans in 2043.
Model breakdown has occurred. (Or has it?)

Every college graduate should . . .

understand that a model is the best approximation available that will likely be modified as more data is collected.

Verify the Fit



Every college graduate should . . .

be able to use models to support an argument.

Use Models to Support an Argument

The declining baseball fan model might be used to persuade . . .

a city to decline to build a new baseball stadium.

a television station to decline airing baseball on their station.

Modeling Illuminates Concepts

- Functions
- Inverse Functions
- Algebra of Functions
- Graphing
- Meaning of Parameters
- Solving Equations
- Model Breakdown
- Critical Thinking
- Rule of Four
- Technology

Song

The Number Guy

Selecting a Model

Year	Federal Cost of Secrecy (billions of dollars)
1997	3.4
1998	
1999	
2000	
2001	
2002	
2003	
2004	

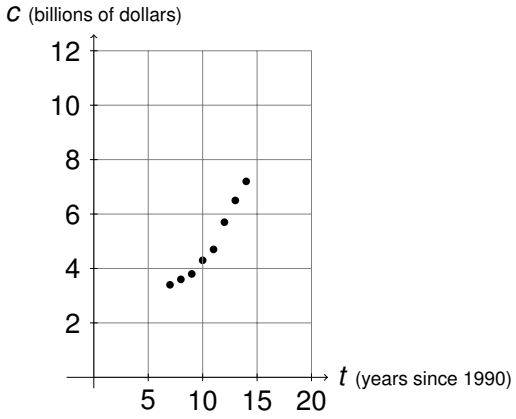
Source: *National Security Archive*

Selecting a Model

Year	Federal Cost of Secrecy (billions of dollars)
1997	3.4
1998	3.6
1999	3.8
2000	4.3
2001	4.7
2002	5.7
2003	6.5
2004	7.2

Source: *National Security Archive*

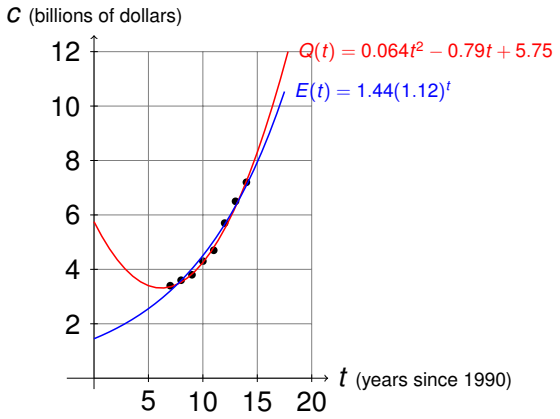
Scattergram: Federal Cost of Secrecy



Every college graduate should . . .

be able to develop a reasonable plan even if an authentic situation is ambiguous.

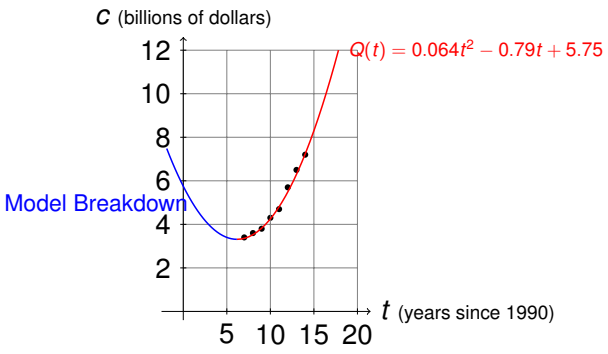
Comparing Mathematical Models



Criteria for selecting a model

- The graph of the model should fit the points well.
- The model should make sense within the context of the authentic situation.
- Piece-wise defined functions

Model Breakdown



Every college graduate should . . .

be able to use terminology such as *linearly*, *exponentially*, *varies directly*, and *varies inversely appropriately*.

Use Correct Terminology

Two theories:

For the years before 1997, federal cost of secrecy and time are approximately exponentially related.

From 1997 on, federal cost of secrecy and time are approximately quadratically related.

Meaning of Parameters

$E(t) = 1.44(1.12)^t$

Meaning of the coefficient 1.44
The cost was \$1.44 billion in 1990.

Meaning of the base 1.12
The cost is increasing by 12% per year.

Estimations and Predictions

Estimate when the cost was \$2 billion.

$$\begin{aligned} 2 &= 1.44(1.12)^t \\ \frac{2}{1.44} &= 1.12^t \\ \log\left(\frac{2}{1.44}\right) &= \log(1.12^t) \\ \log\left(\frac{2}{1.44}\right) &= t\log(1.12) \\ \frac{\log\left(\frac{2}{1.44}\right)}{\log 1.12} &= t \end{aligned}$$

$t \approx 3$ (1993)

Estimations and Predictions

Predict when cost will be \$16 billion.

$$\begin{aligned} 16 &= 0.64t^2 - 0.79t + 5.75 \\ 0 &= 0.64t^2 - 0.79t - 10.25 \\ t &= \frac{-(-0.79) \pm \sqrt{(-0.79)^2 - 4(0.64)(-10.25)}}{2(0.64)} \end{aligned}$$

$t \approx -8$ (1982) and $t \approx 20$ (2010)

Every college graduate should . . .

be fluent in converting units, including units such as thousands, millions, and billions.

Estimations and Predictions

Estimate the cost in 1985.
$E(-5) = 0.64(-5)^2 - 0.79(-5) + 5.75 \approx 0.82$

$0.82 billion
or $820 million

Student Quote

It's a way that I haven't seen before. It makes me see math as a part of my life. It makes it easier to me because I'm more interested. It's very easy to understand because it's real data. You can't fake real data.

Student Quote

Mathematics is numbers and trying to figure out why x should be a number, too, despite the fact that it's quite content as a letter.

Every college graduate should . . .

be able to sift through large amounts of information to perform a task.

Labs

- Pass out samples to students.
- Collect data through experiments or research.
- Find an appropriate model.
- Use the model to make estimates and predictions.
- Students type a report.
  - Opening paragraph
  - Respond to generic questions.
  - Summary paragraph
- Students turn in labs in two stages.
- Use a rubric to grade labs.
- Benefits
  - Students see big picture.
  - Students see that math is useful.
  - Writing across the curriculum

Projects

- Consist of three labs: linear, exponential, and quadratic
- Projects are submitted in six stages.
- Students can resubmit projects.
- Oral presentations
- Benefits
  - Same as for labs
  - Serves as great prep for final

Project Requirements

- Write-ups must be typed.
- Questions for the different types of models are given in the Topic of Your Choice Labs (linear, exponential, and quadratic).
- When finding an equation of an appropriate model:
  - First show a linear, quadratic, and exponential regression equation for each set of data.
  - Then choose the best type of function to use as a model. Explain why it is the best choice.
  - Then find the equation of your chosen type without using regression. (Do it by hand.)
  - Finally, compare the equation you found by hand with the regression equation.
  - Use the regression equation to solve remaining problems.
- You must show your work for *every* problem.

Project Assignment: Linear Part

1. What two quantities did you explore? Define variables for the quantities. Include units in your definitions.
2. Which variable is the dependent variable? Which variable is the independent variable? Explain.
3. Describe how you found your data. If you conducted an experiment, provide a careful description with specific details of how you ran your experiment. If you researched data, state the source of your data.
4. Include a table of your data.

Project Assignment: Linear Part

5. Use a graphing calculator to draw a scattergram of your data. (If your data are not approximately linear, find some data that are.)
6. Find an equation of a linear model to describe the data.
7. What is the slope of your linear model? What does it mean in this situation?
8. Does it make sense that your variables are approximately linearly related in terms of the situation you chose to model? Explain.

Project Assignment: Linear Part

9. Choose a value for your independent variable. On the basis of that chosen value, use your model to find a value for your dependent variable. Describe what your result means in the situation you are modeling.
10. Choose a value for your dependent variable. On the basis of that chosen value, use your model to find a value for your independent variable. Describe what your result means in the situation you are modeling.
11. Find the intercepts of your linear model. What do they mean in the situation you are modeling? Has model breakdown occurred at the intercepts?

Project Assignment: Linear Part

12. Comment on your lab experience.
    1. For example, you might address whether the lab was enjoyable, insightful, and so on.
    2. Were you surprised by any of your findings? If so, which ones?
    3. How would you improve your process for this lab if you were to do it again?
    4. How would you improve your process if you had more time and money?

Rubric

- 5% ____ Each data set contains at least five points.
- 5% ____ Each data set is modeled well by a function (there is good fit).
- 5% ____ For each data set, a source is provided.
- 10% ____ For each data set, scattergrams and three models are shown and there is a complete explanation of which model is best.
- 60% ____ Responses to all questions are correct. All work is shown.
- 10% ____ Project is typed, well-organized, precise graphs, cool cover
- 5% ____ Opening paragraphs and summary paragraphs are thoughtful and complete.

Student Quote

This is without a doubt the best math course I've taken. I've taken this very course three times and have either dropped or failed. This is the only class that I've seen all the information so clearly.

Every college graduate should . . .

know that mathematics is truly useful in the "real world."

Linear Modeling

Table: Ford's U.S. Market Shares

Year	Market Share (%)
1998	26
2000	23
2002	20
2004	19
2006	15
2008	14

Source: *Ward's AutoInfoBank*

Linear Modeling

Table: Percentages of Americans Who Think That the Press Has Too Much Freedom

Year	Percent
2000	51
2001	46
2002	43
2003	46
2004	42
2005	39
2006	40
2007	34

Source: *First Amendment Center*

Linear Modeling

Table: Percentages of Army Recruits Who Needed to Get Conduct Waivers Because of Criminal Records or Other Past Misconducts

Year	Percent
2004	4.6
2005	6.0
2006	7.9
2007	11.2
2008	13.0

Source: *U.S. Army Recruiting Command*

Linear System Modeling

Table: Chrysler and Dodge Sales

	Number of Vehicles Sold (millions)	
Year	Chrysler	Dodge
1998	0.31	1.44
2000	0.47	1.50
2002	0.47	1.26
2004	0.60	1.20
2006	0.61	1.08

Source: *Autodata*

Exponential Modeling

Table: Average Prices of Flat-Panel Plasma Televisions

Year	Average Price (thousands of dollars)
2000	9.8
2001	6.8
2003	4.6
2005	2.5
2006	1.7
2007	1.3

Source: *DisplaySearch*

Exponential Modeling

Table: Percentages of American College Students Who are Minorities

Year	Percent
1976	15
1980	16
1990	20
2000	28
2005	31

Source: *Department of Education*

Exponential Modeling

Table: Amounts of Digital Data Stored in a U.S. Household

Year	Amount of Digital Data Stored on Devices in a Typical U.S. Household (terabytes)
2004	0.4
2005	0.5
2006	1
2007	1.6
2008	2.5

Source: *Coughlin Associates*

Quadratic Modeling

Table: Numbers of International Adoptions in the United States

Year	Number of Adoptions (thousands)
2001	19.3
2002	21.5
2003	21.5
2004	23.0
2005	22.8
2006	20.5
2007	19.3

Source: *U.S. Department of State*

Quadratic Modeling

Table: Percentages of Police Officers Who Are Women

City Size (in thousands)	City Size Used to Represent City Size (in thousands)	Percent
0-9.999	5	8.3
10-24.999	17.5	7.5
25-49.999	37.5	8.5
50-99.999	75	9.4
100-249.999	175	11.7
250 or more	300	17.0

Source: *FBI Uniform Crime Report*

Quadratic Modeling

Table: Fatality Rate Per 100 Tornadoes

Year	Fatality Rate (Number of fatalities per 100 Tornadoes)
1998	18
1999	11
2000	5
2002	3
2004	2
2006	9
2007	11

Source: *National Oceanic and Atmospheric Administration*

Square Root Modeling

Table: Percentages of Foundations That Compensate All of Their Board Members

Asset Group (millions of dollars)	Asset Used to Represent Asset Group (millions of dollars)	Percent
0–5	2.5	4
5–10	7.5	8
10–25	17.5	10
25–50	37.5	15
50–100	75	20
100–250	175	21
250–500	375	32

Source: New York Times

For more information . . .

Balancing Concepts, Skills, and Curve Fitting

- Are there skills or concepts in intermediate algebra that aren't needed for subsequent courses?
- Are there any skills or concepts in intermediate algebra that might be better addressed in subsequent courses?
- Trade some (or all) traditional word problems for some curve fitting.

Elementary Algebra and College Algebra

Curve fitting can be extended to elementary algebra and college algebra.

Elementary algebra: linear and quadratic models

Intermediate algebra: linear, exponential, quadratic, rational, and radical models

College algebra: determining which model to use; piecewise defined models

Student Quote

There is actually no doubt that using actual data not only broadens our understanding of the real world, but it also helps us turn mathematical concepts into tangible and concrete ideas.

For more information . . .

A student should be able to . . .

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