Making Mathematics Meaningful for Calculus Students

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The Need for Meaning in Mathematics Learning

Applications of calculus to engineering, physics, and biology are a well-known and often an important part of the undergraduate calculus curriculum. However, little time is usually spent discussing the many applications of calculus to other fields of study. The American Mathematical Association of Two-Year Colleges (AMATYC) states "mathematics that students study should be meaningful and foster their appreciation of the discipline" (AMATYC, 2006, p. 10). Many students in undergraduate calculus classes are not planning to pursue careers in the few fields in which they will see an application of calculus. Therefore, instructors need to find ways to help students see the usefulness of mathematics for their chosen career path.

Some researchers have found a decrease in perceived usefulness of mathematics among students from 9th to 12th grade (Chouinard & Roy, 2008) while others have identified the assignment of contrived, unrealistic examples (mathematically simplistic enough to avoid confusion) as one reason that students may feel this way (Duan, Depaepe, & Verschaffel, 2011). More meaningful and interesting problems can sometimes require techniques that are beyond students' understanding or are computationally tedious. Therefore, the classroom activity described in this article was designed to give students a vision of the power of mathematics to solve problems of personal interest to them, without having to spend a great deal of class time delving into complicated mathematics. This activity specifically fosters an environment "that optimizes the learning of mathematics for all students" (AMATYC, 2006). In addition, it also serves a secondary purpose: to particularly encourage women interested in mathematics and the mathematical sciences. Research has shown that women tend to want to work with people not things (Lubinski & Benbow, 2006), and that social values are important in choosing a career (Duffy & Sedlacek, 2007). The activity was piloted in a Calculus I class of 24 students at one of the authors' institutions. We later describe some results from the pilot project.

Implementation

The activity, Calculus Everywhere, consists of two parts. First, students are presented in class with examples of how mathematics is applied to different fields. Examples shown to students are from research articles and the intent is to provide students with a connection between the calculus they are learning in the classroom and a variety of issues of social concern. We use the presentation to highlight the mathematics used in the research and also to foster a discussion among students about what majors or careers might make use of this type of research.

For a recent implementation of this activity in a calculus class at one of our institutions, we chose examples relevant to majors many of our students have chosen. Examples on bioterrorism issues (Atkinson & Wein, 2008) were chosen to appeal to the large number of criminology and forensic science majors in the course. We also chose applications of calculus to Africa's economic problems (Sachs, McArthur, Schmidt-Traub, & Kruk, 2004) because they very clearly addressed a significant social problem. Voting applications (Gelman, Katz, & Tuerlinckx, 2002) were particularly timely given the recent 2012 presidential election.

The second part of the activity is an out-of-class homework assignment. For the assignment, students were asked to first locate a research article that applies calculus to either their academic major or their intended career field. Once they located an article they were to write a half-page summary, making sure to note the use of mathematics, as well as the kind of major or career that may be affected by the problem addressed in the article. See Appendix A for the assignment and grading rubric we use.

Student Difficulties

For the out-of-class assignment, we ask students to e-mail their instructor a copy of their article and a one-sentence description of the mathematics in it. We have found that requiring this before they start writing the summary is helpful in combating student difficulties. Given that in the first

implementation of this activity several students had difficulty finding an appropriate article to use for this assignment on their first try, correspondence with the instructor was helpful for evaluating the article for relevance. Difficulties encountered by students included the assumption that summary papers (found online) written by other students, blog posts, or instructor-created worksheets with different application problems (also found online) were appropriate sources for the assignment. In addition, since many of our Calculus I students were undergraduate freshmen with less experience navigating our library search services, it was usually a challenge for students to find research literature. Instructors were able to redirect these students to appropriate searches or sources. Despite these problems most of our students have been able to find relevant articles and write thoughtful summaries of the mathematical applications.

Our Students' Reactions

Our aim with this activity was to increase our students' awareness of the usefulness of mathematics in society. The following student comments were gathered during one semester of implementation from their summaries that describe their attitudes, appreciation for the value of mathematics, and some of their choices to investigate mathematics that applies to social problems. These sample responses represent themes that were common in the students' essays.

Attitude

The majority of students who made statements in their summaries about the societal value of mathematics demonstrated an increased awareness of this value. Two students' comments indicated dramatic shifts in perspective from believing that calculus was pointless for their future careers to seeing directly how it was useful to them. The first student chose an article that discussed using technology in the biology classroom to explore famous calculus-based mathematical models such as Fischer's Fundamental Theorem of Natural Selection (Jungck, 2010). This article helped the student to see why calculus was important for her major: "I am a biochemistry major and I'll admit that at first I had trouble understanding why Calculus 1 and Calculus 2 were required for my major. The article I found discussed 10 equations that have changed the way biology, as a science, operates."

A biology major considered a paper that described a fractional-order model of neurons in the body (Debnath, 2003). She says, "the part of the paper that actually interests me is … the use of calculus in biology, which I never thought was possible… ." Her essay reveals she has discovered the importance of mathematics to furthering research in her anticipated career field and, more importantly, that she was unaware of this fact before the Calculus Everywhere activity.

Appreciation

A student planning to become an oncologist stated that without Maxwell's equations "MRI's wouldn't be possible and [her] future career would be very difficult." Another, after reading an article comparing the rate at which a drug left the body to the rate that it is taken in to create a steady state (Kale, Lohiya, Rasala, & Avari, 2010), determined that calculus is taught to aspiring doctors and potential medical researchers because it is "essential in understanding the line of work and to be able to make progressive strides in the field."

Addressing Social Problems

Some students chose topics that had direct impact on social problems. A forensics major chose an article that discussed a more effective way of determining the concentration of illicit drugs seized from the streets. One general-studies major read a paper describing applications of a spatial analysis model used by people in geographical studies to study patterns among humans in different environments. She added that "the model [could] also be used by people involved in global studies because it [allowed for] the study of the impact of possible scenarios on the lives and economic prosperity of the people in whatever region of the world they [were] studying." She also describes how this article would be useful to computer scientists and economists.

Additional Data on Student Attitudes

In addition to essays from students that provided us with data from their personal experiences with the Calculus Everywhere activity, we used the Attitude Towards Mathematics Inventory (ATMI) survey (Tapia & Marsh, 2004) to collect data on student attitudes at the beginning and end of the semester in which we implemented the activity. ATMI survey questions address one of four different factors: self-confidence, value of mathematics, enjoyment of mathematics, and motivation. We were specifically interested in student responses to questions about the value of mathematics. Data from several value of mathematics survey items are presented in the following tables. The data were collected from the Calculus I class with 24 students.

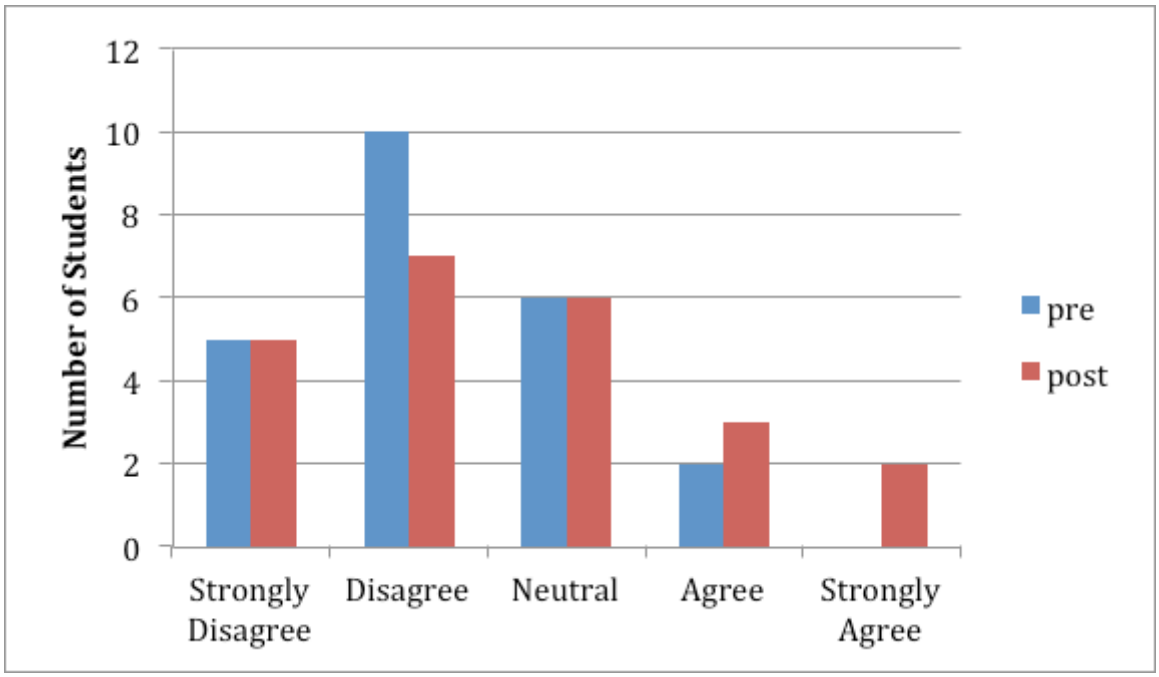

Table 1. **Student Responses to 'I think studying advanced mathematics is useful.'**

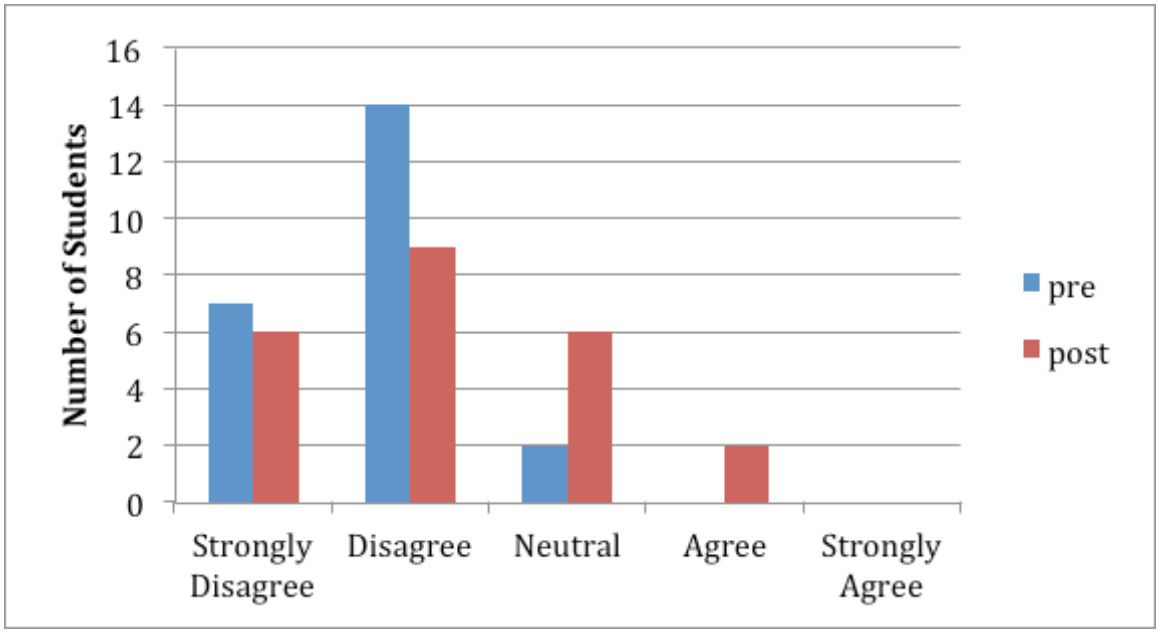

Table 2. **Student Responses to 'A strong math background could help me in my professional life.'**

Overall, there was low belief at the outset of the semester among students that mathematics is useful (Table 1) and that studying mathematics would be useful for problem solving in other areas (Table 3). However, the strongest negative responses from students were with respect to mathematics being useful in their professional life (Table 2). None of the students surveyed believed that mathematics would be useful in their professional life at the beginning of the semester. While the survey data at the beginning of the semester described negative attitudes toward the usefulness of mathematics among our students, the results from the postsemester survey indicate a shift in student perspective with more students responding positively about all three *value of mathematics* items.

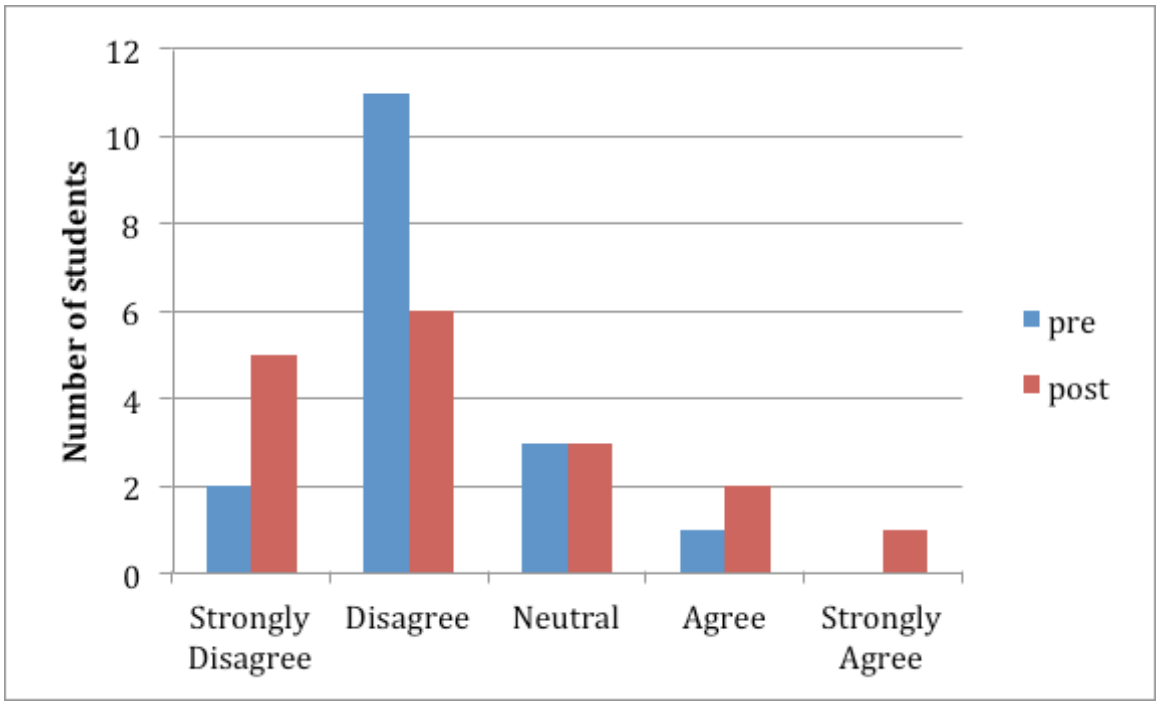

Table 3. **Student Responses to 'I believe studying math helps me with problem solving in other areas.'**

Discussion

Following its implementation, this activity has evolved over several semesters. Specifically, in the first semester, we asked students to summarize the literature they read without much guidance for the summary. It became clear after the first semester that we needed to be more explicit about the content of the essay in order to understand the perceived usefulness of mathematics our students had before they completed the activity, and also to determine if the activity alone had an impact on their beliefs. In addition, we decided to let this assignment be worth twice as much as a regular homework assignment for grading purposes. The idea was that students would take the assignment more seriously if it had a greater effect on their overall grade. Subsequently, in dealing with both of these issues, we now provide students with a grading rubric that is explicit on exactly what questions need to be answered in an essay—in addition to describing the mathematics that is used—and how the summary will be graded. The revised essay directions include requiring students to answer questions such as "What career field might make use of this application of calculus?" and "Did you know this before reading the article?" We continue to improve this activity based on student feedback and results of our own analysis.

While our implementation occurred in a Calculus I class, a similar activity can be done at any level, with modifications. Students in classes beyond calculus should require little in the way of modifications and will hopefully have fewer problems with logistics such as using library resources for literature searches. Students in algebra and precalculus may have difficulties finding research that uses the mathematics they are learning. However, one aspect of the Calculus Everywhere activity that made it doable for students was that they were not required to understand the mathematics (calculus) in their research papers. They simply had to observe calculus being applied to another field and summarize its use. So, a modification for students in lower-level mathematics classes could be as follows: (a) instructors can base their presentations off of articles that use the appropriate level of mathematics or (b) have students instead read articles from undergraduate research journals that may be at a more appropriate level. For high school students, it might be more suitable to ask students to find a news article or other source instead of a research paper. Also, incorporating more frequent in-class discussions in lieu of out-of-class assignments would allow the instructor to present the activity at the proper level as often as time allows. Instructors who wish to emphasize mathematical communication through this assignment can use articles as the basis for an in-class poster presentation (Brown & Burroughs, 2008).

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Appendix A

Everyday Calculus – Part II: Homework Assignment

Your assignment is to find an application of calculus to your major or another area that you find interesting. This article should be a research article, not just a blog or random website. The first part of the assignment is to e-mail me with the topic you have chosen and the article you are using. This is due by **[due date]** although you are encouraged to do so earlier. I will let you know if this is an appropriate topic/article.

Then, for the second part of the assignment, turn in a half-page essay (12-point font, single-spaced) summarizing the article, mentioning the kind of mathematics used, as well as what kind of major or career you might have to make use of this information. Please attach a copy of the article to this essay. This is due **[due date]**. You must also answer the questions below.

Directions: Your essay will be graded based on this rubric. Consequently, use this rubric as a guide when writing your essay and check it again before you submit your essay.

Circle the word that best describes your attitude toward each statement.

1. I think studying advanced mathematics is useful.
   Strongly Disagree Disagree Neutral Agree Strongly Agree
2. I believe studying math helps me with problem solving in other areas.
   Strongly Disagree Disagree Neutral Agree Strongly Agree
3. A strong math background could help me in my professional life.
   Strongly Disagree Disagree Neutral Agree Strongly Agree

Provide a paragraph response to the following questions:
What future job or career do you plan to have? How important is mathematics for this area and why?

Grading Criteria:

(0) Poor
(1) Below Average
(2) Average
(3) Good
(4) Excellent

Research Based	(0) The essay is based on an article that does not use calculus. (1) The essay is based on a website, blog, or worksheet that uses calculus. (2) The essay is based on a research article that loosely uses calculus. (3) The essay is based on a research article that clearly uses calculus. (4) The essay is based on an interesting research article that clearly uses calculus to solve a worthwhile problem.
Content	
How is calculus used?	(0) Did not answer question. (1) Answer is partial or incomplete. (2) Answer is not comprehensive or completely stated. (3) Answer is accurate and complete. (4) Answer is comprehensive, accurate, and complete.
Did you know this before reading the article?	(0) Did not answer question. (1) Answer is partial or incomplete. (4) Answer is complete.
Organization	(0) The essay is unclear with no organization. (1) Organization and structure detract from the answer. (2) Inadequate organization or development. Structure of the answer is not easy to follow. (3) Writing follows a logical organization but sometimes drifts from the main point. (4) Writing is clear, logical, andorganized around a developed thesis.
Grammar, Mechanics, and Spelling	(0) Sentences are awkward, repetitive, or difficult to understand. The author makes numerous errors that interfere with understanding. (1) The author makes several errors in grammar, mechanics, and/or spelling that interfere with understanding. (2) The author makes several errors in grammar, mechanics, and/or spelling but they do not interfere with understanding. (3) The author makes minor errors in grammar, mechanics, and/or spelling, but they do not interfere with understanding. (4) The author makes no errors in grammar, mechanics, and/or spelling.