Challenging Mathematically Gifted Students in Elementary CGI Classrooms
*(CGI: Cognitively Guided Instruction)*

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First, let’s have a quick “CGI experience”!

Here is a typical problem posed in a 5th grade CGI math lesson.
Solve this problem in any way that makes sense to you:

Sara has 12 cups of cat food. She gives her cat 1½ cups of food each day.

How many days of cat food does she have?
Overview of presentation

• What is mathematical giftedness (MG)?
• Common approaches to challenging MG children at the elementary school
• What is “CGI” approach to teaching math?
• Teacher strategies for challenging within a CGI math lesson
• Discuss the pros and cons of CGI as a an approach to challenging mathematically gifted students in a mainstream classroom
Recognizing mathematical giftedness

• Keen awareness and curiosity for numbers
• Quickness in learning, understanding, and applying mathematical ideas
• Intuitive sense, often answering problems without awareness of a sequence of steps
• High ability for abstract thinking
• High spatial reasoning ability
• Positive, motivated attitude towards mathematics and high self-efficacy
Recognizing mathematical giftedness

- Flexibility and creativity in problem-solving
- Persistence in problem-solving, knowing when to re-direct
- High ability for seeing mathematical relationships and patterns and making generalizations
- High ability for transferring learning to new, untaught mathematics without a teacher’s guidance
Percentage of Mathematically Gifted in our Population?

• The “experts” don’t want to commit to a number!
• Depends on our definition of mathematically gifted… 1/600 or 1/25?
• If we look at how many students end up in Calculus BC in high school… 1-4% 
• Johns Hopkins Center for Talented Youth… Above-level testing in elementary grades indicates 1-3% of students capable of above-level mathematics
The Way it is…

• Most mathematically gifted elementary students are taught math in mainstream classrooms in which teachers are responsible for students with a range of mathematical abilities.

• Most of these mathematically gifted students are not being adequately challenged. (Assouline & Lupkowski-Shoplik, 2005).
Preaching to the Choir…

Mathematically gifted students need and deserve the opportunity to work to their potential…

Mathematically gifted students need to be challenged on a daily basis, avoiding repetitive work that they have already mastered (which could lead to boredom and decreased interest in mathematics).
Research supports *acceleration* for students capable of working 1-2 grade levels ahead.

A Nation Deceived: How Schools hold back America’s Brightest Students
Coangelo, Assouline, & Gross (2004)

Johns Hopkins Center for Talented Youth and other talent searches...


**Recommendation:** Mathematically gifted students with sufficient motivation appear to be able to learn mathematics much faster than students proceeding through the curriculum at a normal pace, with no harm to their learning, and should be allowed to do so.
Yet it remains…

Most elementary mathematically gifted students are taught in a mainstream classroom.

Thus, it’s sensible to consider how to maximize the mathematically gifted students’ experiences in the regular classroom.
Some Approaches…

Within Class:
- Enrichment and Differentiated Instruction at grade level (*depth and complexity without advancing to higher math, fewer problems needed to demonstrate mastery*)
- Clustering
- Individualized paced instruction, advancing to higher math, but staying with grade level teacher (*telescoping, curriculum compacting, DT → PI*)

Outside of Class:
- Mentorship
- Distance Learning (enrichment or advancement)
- Math Clubs, Contests
Some cautions…

• Implementation of these ideas are sporadic as most mg students say they are rarely challenged.

• Enrichment may not be enough…
  Although mathematically gifted children tend to enjoy math enrichment topics, they typically want to and need to tackle advanced math topics. Catering to their needs will usually involve a combination of enrichment and acceleration.

• Beware of isolating the student in this intellectual pursuit…Provide opportunities for mathematical communication with intellectual peers and mentors.
Linda Sheffield, editor of NCTM’s *Developing Mathematical Promise* (1999), suggests…

A classroom environment that offers an open-ended problem solving approach, with opportunities for discovery and experiential learning may allow mathematically gifted students to…

- Learn at a faster pace
- Explore mathematics with more breadth and depth
Cognitively Guided Instruction (CGI) utilizes such an open-ended problem-solving approach to teaching mathematics.
A typical CGI math lesson in a *mainstream* class:

- **Phase 1:** Teacher presents a problem to the class
  
  *Example:* Sara has 12 cups of cat food. She feeds her cat 1 ½ cups of food each day. How many days of cat food does she have.

- **Phase 2:** Students are given ample time (15-25 minutes) to solve this problem in whatever ways make sense to them, “inventing” their own strategies, and justifying their solutions. Teachers encourage students to find more than one strategy for solving the problem and to communicate with other students. Sometimes, extra problems are introduced for those who finish early.

- **Phase 3:** The problem-solving experience culminates in the student-sharing phase (approx. 15-25 minutes) in which a variety of strategies (carefully selected by teacher) are presented by students for class discussion. Teachers help students see the main concepts and make generalizations.
Synonyms for CGI …

• “Problem-based learning”
• “Student-centered”
• “Reform-oriented” math instruction – fits in well with Common Core Math Standards

A classroom doesn’t have to claim “CGI” status to be using the CGI philosophy toward math instruction. A big difference, however, is the teachers of CGI classrooms have undergone professional development (up to 3 years) that enhances this approach.
The Origins of Cognitively Guided Instruction (CGI)

• University of Wisconsin research on children’s understanding of mathematics (1980’s)

• Predicted that student achievement would increase if teachers better understood their students’ mathematical thinking.

• A professional development program to increase teachers’ understanding of student thinking, termed Cognitively Guided Instruction, was conducted for 1st grade teachers as part of an experimental study.

• Classroom implementation of the CGI approach led to increased student achievement, confirming the prediction. (Carpenter et al., 1989)

• In “CGI Classrooms,” teachers guide their instruction based on their knowledge of how their students think about mathematics.
CGI Professional Development

• CGI PD primarily involves analyzing student problem-solving strategies to deepen teachers’ knowledge of how students think about mathematics.
• It is emphasized that teachers must have strong mathematical content knowledge in order to understand their students’ mathematical thinking.
A CGI Math Lesson in an Elementary Math Class

• Constructivist in nature, allowing students to solve math problems in ways that make sense to them
• Problem-solving based, encouraging multiple strategies and mathematical discourse
• Equity-oriented, respectful of all students’ strategies
Research has consistently shown...

- Teacher confidence and teacher knowledge of their students’ mathematical thinking has increased as a result of CGI professional development.
- Student achievement increases as teacher knowledge of their students’ mathematical thinking grows.
- Students’ problem solving skills increase in CGI classrooms.
- Subgroups of low ability math students, learning disabled students, and English language learners show particular benefits.
The focus of my current research…

Investigating the experiences of mathematically gifted students in CGI classrooms to find out

1) to what extent and in what ways are they being challenged mathematically?

2) what strategies do teachers use to challenge these students?
CGI Teachers say…

• “Gifted kids love CGI math.” (compared to a traditional worksheet-driven classroom, this is understandable)
• “The focus on ‘invented strategies’ and flexible thinking is right up their alley as creative thinkers.”
• “They like to talk about their strategies on the board.”
• “They spring into action when I ask them to justify their answers.”
• “They like discussing and critiquing other students’ strategies.”
What I initially questioned about CGI and the mathematically gifted…

• Ample time for an average math student (or low student) is quite different than ample time for a gifted math student
• How is this reconciled when you have all ability levels in a mainstream elementary classroom?
• With understanding students’ thinking at the heart of CGI, do some teachers find it difficult to follow the advanced thinking of MG students?
• How can a MG student possibly advance when the class does 1 or 2 problems a day? (many CGI classes have 2 math times, one for problem-solving and one for more content-driven instruction. A MG student deserves to be challenged both times…)
In particular…

What happens when the gifted student is done with the problem in 1 minute and other students continue to work for 20 more minutes… (does a MG student ever work for the entire time?)

Do the problems posed allow the student to exhibit the characteristics of mathematical giftedness?

What strategies does the teacher use to try to keep the MG student engaged and challenged?
On one end of the spectrum …

- A 4th grade CGI class is asked:
  
  *There are 10 bags of candy with 13 candies in each bag. How many candies total?*

- MG student quickly shows 3 strategies then reads a book for 15 minutes while other students finish.

- When interviewed later about the easiest way to solve the problem, he replies, “I just knew the answer. It’s mental math.”
On the other end of the spectrum...

- Even though the student answered the “cat food” problem in less than a minute, he stayed after class to talk to the teacher announcing that he thinks he sees why you multiply by the reciprocal when dividing by a fraction.

- The catalyst for his thinking had been another students’ pictorial solution that had been shared on the board (the other student was not gifted).
Problem: 12 cups of cat food. 1/2 cups per day. How many days?

Solution by an "average" student:
Solution by an average student:

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<th>5 days</th>
<th>7 days</th>
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Solutions by a Gifted Student:

1st strategy

\[
\frac{1\frac{1}{2}}{3} \times 2 \rightarrow \frac{12}{3} \div 3 \rightarrow 2 \text{ days} = 3 \text{ cups} \rightarrow 4 \times 2 = 8 \text{ days}
\]

2nd strategy

\[
12 \div 1\frac{1}{2} = 12 \div \frac{3}{2} = 12 \times \frac{2}{3} = \frac{24}{3} = 8 \text{ days}
\]

Later: an AHA Moment!!!
Phase 1 of a CGI lesson: The problem posed

- Should have multiple levels
- Should allow MG students to reach a little further (zone of proximal development)
- Yet if the mg student is given a problem much different than the rest of the class, it is often not discussed in the strategy sharing phase *(what to do about that?)*
- Could present a series of problems to the MG student that would serve as a trajectory for a generalization to be made. This generalization could be shared with the class.
Example of a problem that could lead to a generalization…

Problem posed:

___ kids are sharing _____ sub sandwiches. If they share equally, how much will each kid receive?

(3,4)  (3,8)  (6,20)  (12,40)  (6,10)

The gifted student’s quick response:

4/3  8/3  20/6  40/12  10/6
Re-wording the question to press connections…

___ kids are sharing _____ sub sandwiches. If they share equally, how much will each kid receive?

(3,4) (3,8) (6,20) (12,40) (6,10)

Ask: How could you use one of your answers to figure out the answer to another set of numbers?
A common strategy that CGI teachers use to challenge their gifted students...

1. **Use bigger numbers in the problem**

   **Example A:** There are 27 buttons in one jar and 35 buttons in the other jar. How many buttons are there in all? (or use 278 and 359)

   **Example B:** Joe ate \( \frac{1}{2} \) a candy bar and Jill ate \( \frac{1}{2} \) of a candy bar. How much did they eat total? (1/2, ¼) (1/2, 1/8) (2 ½, 3 ¼) (4 ½, 3 1/3)

   (but does this extend the student’s thinking or just make the problem take longer to do?)
Change the numbers to increase the challenge…

**Example C:**

This is $\frac{1}{2}$ of the cookies I baked. How many did I bake? (try replacing with $\frac{3}{10}$…)

[6 filled circles]
Changing the numbers to increase the challenge…

Example C:

This is $\frac{1}{2}$ of the cookies I baked. How many did I bake? (try replacing with $\frac{3}{10}$…)

The solution requires “unit-izing” to $\frac{1}{10}$…
Strategies used by teachers in Phase 2: Problem-solving...

1. Encourage students to provide multiple strategies (yet remind them that no one kind of strategy is being expected...)

This is a good idea if the problem is complex enough to elicit more creative strategies. If the problem is not complex to the MG student, the student should not have to spend time providing more than 1 or 2 strategies to do a simple problem. (i.e. 10 bags of 13 candies)
Strategies used by teachers to challenge…

2. Give more problems (o.k. if there is something unique about the new problems)

Example 1: Teacher purposely jumped from “7 candy bars shared 4 ways” to “25 shared 2 ways” to see if they saw groups of 10’s right away
Strategies used by teachers to challenge...

- **Example 2:** Class had a story problem of 42 kids, tables of 8, how many tables needed? (answer is 6)

- Extra problem said to write a new story where answer to 42 divided by 8 is 5, or 5 ¼.
Strategies used by teachers to challenge…

- Prompt MG students to see the underlying properties within their invented strategies (which encourages the use of their advanced abilities to generalize and may lead to discovery of common algorithms)

Example of solution using distributive property:

6 people each ate 1 ½ pieces of cake. How much cake was eaten?

“Distribute” the 6: \[6 \times 1 + 6 \times \frac{1}{2} = 6 \times (1 + \frac{1}{2})\]

Also recognize same as \(6 \times \frac{3}{2}\) …
Phase 3: Sharing Strategies

- MG students who finished early and appeared bored, for the most part, spring back to life for this culminating part of the lesson. I have observed:
  - They point out mistakes or improvements on other strategies
  - They make comparisons of how their own strategy differs
  - They respond to teacher questions that elicit understanding of generalizations
  - They point out unique ideas, connections
The Potential Advantages

• The time allotted to problem-solving, mathematical thinking, and justifying or “proving” solutions

• CGI is known for introducing problems before grade level standards would otherwise dictate since a direct modeling approach can often solve the problems without being taught an algorithm

Example: 19 children are taking a mini-bus to the zoo. They will have to sit either 2 or 3 to a seat. The bus has 7 seats. How many children will have to sit three to a seat, and how many can sit two to a seat? (a study of kindergarteners showed half of them could solve)

• The opportunity for social interaction/mathematical communication that could deepen mathematical understanding

• The classroom climate that respects the idea that students have different ways of making sense of a problem and thus different strategies (good for the gifted kids to not always be the “star of the show” and good, to some extent, for them to see other children’s solutions)
Benefits observed…

• Mathematically gifted students who are often known to get an answer mentally but not be able to describe it learn to explain their solutions on paper and verbalize them to the class.

• Students may use other students’ shared strategies and class discourse as a springboard for further investigation.
Some concerns...

- Can a mg advance when only 1 or 2 problems are done each day?
- In an open-ended problem-solving environment, the MG student may present a solution that the teacher does not understand.
- The ability of the teacher to extend the mathematically gifted students’ thinking depends on not only content knowledge but the teachers’ flexibility of thinking (for which CGI trains).
What can be done to maximize the teacher’s ability to challenge students in a CGI math class?

• Content knowledge
• Knowledge of how students think about mathematics, with attention to flexible thinking
• Knowledge of characteristics of mathematically gifted children and the way they think
My hypotheses so far...

Compared to a traditional mainstream elementary math class in which the teacher does not differentiate instruction, a CGI math class in some ways allows for the MG students to differentiate for themselves.

Compared to a teacher-centered, skill-oriented math class, the MG student has more opportunity to exhibit mathematical gift in a CGI problem-solving environment. However, the potential for wasted time, lack of enrichment/advancement exists. The mg experience will depend on the teacher posing adequately challenging problems and having adequate knowledge and attentiveness to extend the students’ thinking.
Recommendations for CGI Classrooms

• Cluster the brightest of the brightest math students for each grade level (Winebrenner)

• Have this class taught by the teacher who has the greatest mathematical content knowledge and interest (essential for asking questions of appropriate depth and complexity)

• Both of the above will increase the challenge of the mathematical discourse, allowing more connections to be made, etc.
Decreasing the Range of Mathematics Instruction when “Clustering”

Clustering

• The “cluster” of mathematically talented children makes it more likely that the teacher will differentiate for their needs
• The decreased range of abilities (average to way above average) makes it easier for the teacher to focus instruction
• The “cluster” provides an intellectual peer group of children who share the gift of mathematical ability (a gifted student working in isolation is not recommended)
More recommendations…

- “Extra” problems posed should extend the students’ thinking beyond the original problem and allow opportunities for the student to make connections and generalizations.

- Teachers should provide individual feedback if the MG solution is not discussed in class.

- Teachers should have occasional one-on-one conferences with MG students to focus on their growth as problem-solvers and mathematical thinkers and to scaffold their metacognition.
CGI Resources

• *Children’s Mathematics: Cognitively Guided Instruction* (1999)
  Carpenter, Fennema, Franke, Levi, & Empson

  Carpenter, Franke, & Levi

  Empson & Levi
Developing Math Talent: A Guide for Educating Gifted and Advanced Learners in Math
Assouline & Lupkowski-Shoplik (2005)

Developing Mathematically Promising Students
Sheffield (1999)
Some Materials for an Enriched Curriculum

Project M$^3$: Mentoring Mathematical Minds is a research-based mathematics program for gifted children in grades 3, 4, and 5. (Kendall Hunt Publishers)

- Challenging and motivational curriculum
- Incorporates the NCTM content and process standards
- Recommended by NAGC as exemplary practice

Note: it is still not a complete curriculum that includes all grade level math standards, but it adds the element of critical thinking and additional challenge to most standards-based concepts.
Other resources...

- *The Elements of Creativity and Giftedness in Mathematics*  
  (Leiken, Berman, & Koichu, 2009)
- *Creativity in Mathematics and the Education of Gifted Students*  
  (Sriraman & Lee, 2011)
- Lindalevi.blogspot.com (CGI)
- hoagiesgifted.org/math_gifted.htm
Please email me your experiences with mathematically gifted students in a CGI classroom!

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