# SQUASH BALL HEATING

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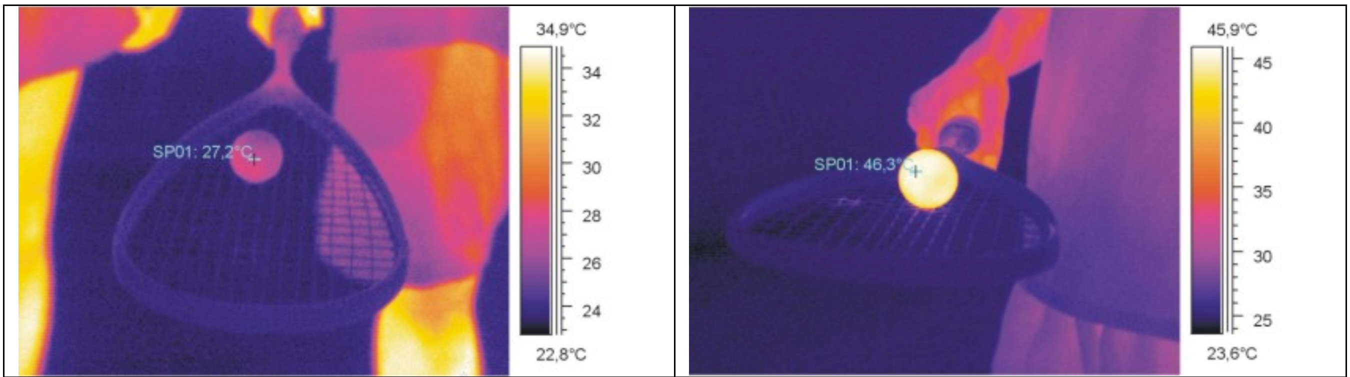
## 1 Preliminary Remarks about Teaching Aspects

In the sense of MUSE, there are two main “More Understanding” aspects aimed at with this contribution:
a) order-of-magnitude reasoning/estimation (OMR), here in order to understand a thermographic image;
b) experimental / observational competences, in particular in order to get or check necessary data.
Both aspects can be seen as components of an understanding of “Nature of Science (NOS)”, according to current convictions in the field an important objective of science (physics) education. The idea is, of course, not to “preach” nature of science, exactly as one would not “preach” science (physics) itself, but to let learners work on and discover it in concrete activities.

## 2 Problem Statement

Have a look at the thermographic images of a squash ball below.
a) Give a qualitative explanation of the temperature increase visible in the images below. State your explanation in terms of an energetically closed system (a system large enough to be considered to stay at constant total energy) and the subsystems and energy exchanges relevant for the problem.
b) Determine the value of temperature increase from the images, and try to explain its order of magnitude.
c) For the following key factors of the process, give an easy and approximate measurement method, possible with means from everyday life: ratio of velocities $v_a / v_b$ before and after the collision; number of collisions per minute.
d) Include a critical consideration of the assumptions you made and the final estimate you obtained (plausibility, neglected factors, limitations); in particular, give the direction in which possible corrections work. Again, try also to give an order-of-magnitude of corrections where easily possible.
e) State anything else which could deserve attention (in the 2nd image)?




Heating of a squash ball after some minute of playing
(M. Vollmer, http://www.fh-brandenburg.de/~piweb/projekte/thermo_galerie.html)

## 3 Qualitative and Order-of –Magnitude analysis

**a), b) Qualitative explanation and order of magnitude of the temperature rise**

The closed system for the problem setting (“closed” in the sense of the problem statement, see 2a) above) is the entire squash court (rest system) comprising ball, walls (floor), racquet, player and air mass in the court (which is a closed hall):
system (S) = ball (B) + walls/floor (W) + racquet (R) + player (P) + air (A).
The basic explanation is of course based on energy conservation: the thermal energy produced is equal to a certain amount (to be determined) of kinetic energy converted during a collision (called “converted energy”, for short”); in order to avoid misunderstandings, 'thermal energy" is of course nothing else than microscopic kinetic energy.
Beyond this basic process, some other forms of energy transfer within S are relevant, e.g. the P-R-B sequence, where the player “pumps” kinetic energy in the ball (originating of course from chemical energy in his muscles). Other exchanges will be considered in the reasoning that follows.
The converted kinetic energy is given by, in turn, is determined by the velocities $v_{b/a}$ before and after the collision. A useful and current quantity in this respect is the “coefficient of restitution” for vertical impact (i.e. orthogonal to the collision surface, which we will assume for OMR purposes throughout),
(1) $k = v_a / v_b$ .

One then has
(2) $\Delta K = ½\, M\, (v_b^2 - v_a^2) = ½\, M\, v_b^2\, (1-k^2) = (1-k^2) K_b$
where $K_b = ½\, M\, v^2$ kinetic energy before hit and $M$ is mass of the ball.
Note, however, that a part of this energy will be transferred to the other collision partner (racquet, wall), and thus only a fraction of $\Delta K$ (denoted by $\alpha$, a kind of partition coefficient for the converted energy) to the ball. One has $\alpha \geq ½$, if the surface hit is at least as elastic as the ball. The next step is equating with thermal energy $Q = C\, M \Delta T$ where $C$ = (specific) heat capacity and $\Delta T$ = temperature rise (in order to avoid misunderstandings, we understand thermal energy as microscopic kinetic energy). One then has
(3) $Q = \alpha\, \Delta K$ or $C\, M_B \Delta = ½ \alpha (1-k^2) M v^2$
Solving for $\Delta T$ yields
(4) $\Delta T = \alpha(1-k^2) K_b / (C\, M_B) = ½\, \alpha (1-k^2)\, v^2 / C$
Note that the three parameters $v^2$, $k^2$ and $\alpha$ (impact speed, restitution coefficient, partition coefficient) describe the collision, where $C$ describes the ball (specific heat capacity). Here, we will use values for the B-W collisions, see below for taking account of the differences for the B-R collisions. Values can be easily obtained from public sources (e.g. wikipedia) or from the literature given below:
$k \approx 0.5$ [LAG11]
$v \gtrsim 20$ m/s
$C$ = 2 … 3 kJ/(kg K) for organic substances, leading to
(5) $\Delta T \gtrsim 0.03$ K.
This value is the temperature rise *per hit*, so a total temperature rise in the order of 10 K is possible after several 100 hits (some minutes of play, see estimate below).

**c) Experimental estimations**

The ratio of velocities $v_a / v_b$ before and after the collision can be obtained by measuring the successive heights of a bouncing ball. One has, again by energy conservation
$v_a / v_b = (h_a/h_b)^{1/2}$
The number of collisions per minute can easily be obtained by watching a squash match; thanks to Youtube that it is possible at any time. The value obtained in this way was 40 collisions per minute (with wall and rackets).

**d) Critical considerations**

On the overall level, the estimate obtained in a) is a 10 K rise after 10 minutes of play, which is the right order of magnitude, but falls short of the actual value of 20 K reported, and "visible" from the image given above (moreover obtained in a shorter period of "some minutes" of play [VM11]). ).
A first correction comes from the velocities, $v \approx 20$ m/s is a considerable underestimation, values up to 250 km/h (70 m/s) are reported. With equation (4), changing $v$ to $v^*$ changes $\Delta T$ to $\Delta T^*$ as
(6) $\Delta T^* = (v^* / v)^2 \Delta T$
i.e. roughly doubling when going from 20 m/s to 30 m/s (a value still accessible for an amateur).
A second correction comes from the value used for the specific heat capacity. The value for organic substances ($C$ = 2 … 3 kJ/(kg K)) is ok for OMR, because it is plausible and easily available. But the actual value for rubber is 1.25 kJ/(kg K) [TET14], and from eq. (4) then follows an increase of another factor of 2.
Last but not least, a factor not considered here are of course radiative and air (convection) cooling. As both increase with increasing $\Delta T$, there will be an equilibrium temperature, where heat transfer by these cooling processes and heat produced by the inelastic collisions are balanced. Again, the equilibrium temperature can be estimated, but this goes beyond the present contribution. The OMR presented here is thus strictly speaking only about whether the converted kinetic energy is sufficient for he observed temperature rise, and cannot pretend to offer a complete model of the situation, in particular including cooling.

Another point deserves proper consideration: The conditions of the wall and racquet collisions of the ball are not the same, and a very rough way of taking account of this would be to ignore the racquet collisions, to divide the number of hits by 2, which does not change the order-of-magnitude of the estimate (4). But it is interesting to look at this more in detail. One can expect differences for all collision parameters in eq. (4), i.e. restitution coefficient, partition coefficient, and impact speed, which we discuss in turn:
*Restitution coefficient (k)*: Compared to the wall, the racquet is much less massive and much more flexible (e.g. oscillations of string membrane and frame), thus new degrees of freedom for energy conversion (as regarding kinetic energy of the ball) come into play. The restitution coefficient will therefore be smaller, but it is sound to assume that it will not change by an order of magnitude; for a little experiment, compare the rebound heights of a ball from a concrete floor and a (hand-held!) racquet.
*Partition coefficient ($\alpha$)*: As just stated, the racquet offers more degrees of freedom for energy conversion than the wall, but of course, the corresponding amounts of energy will stay in the racquet and in the ball. Thus also $\alpha$

will be somewhat smaller, but there is no reason to expect that the elasticity of the ball and the racquet are different by an order of magnitude, thus $\alpha \approx$ ½ is still appropriate for estimation purposes.

*Impact velocity* ($v^2$): Note that we have to consider the relative velocity, i.e. for the racquet collision we have to add its velocity to that of the ball (still under the simplifying assumption of vertical impact). To get an estimate of racquet speed, consider a serve stroke, where it will be roughly the same speed as that of the ball, which for the average serve of a professional player is around 40 m/s; similar values of racquet speeds for other power strokes are reported. A more conservative estimate would estimate racquet speeds from the slower (amateur) ball speed of 20 m/s given above. Relative impact velocity will then vary between 40 m/s and 60 m/s, by virtue of eq. (5) leading to an increase by a roughly factor of 5 to 10. Thus, there is a physical reason for a substantial increase for this factor, due to the square dependence on velocity, while the other two factors are a little smaller, but not by an order of magnitude.

In sum, the differences between wall and racquet collisions will change the estimate (4) by less then an order of magnitude, and in total rather amount to an increase than to a decrease.

**e) Additional observation(s)**

On the right image above, there is a very faint brighter spot on the racquet (to the left of the ball). This might be due to he last impact of the ball on the racquet, and a trace of the energy exchange B-R. Note that [VM11] indeed shows an thermographic image of such an „impact heating“ (on the floor, not racquet, however).

The forearm of the player seems also to display a temperature rise; this seems curious and might deserve further investigation. A possible reason could be that during physical activity, muscles produce heat, which locally can lead to a temperature increase. But this is already another activity.

## 4 Sources

1. LAG11 The dynamic behavior of squash balls, Gareth J. Lewis, J. Cris Arnold, and Iwan W. Griffiths. Am. J. Phys. 79, 291 (2011)
2. VM11 Von Bällen und Schlägern, M. Vollmer, K.-P. Möllmann, Physik in unserer Zeit, 42/4, 202-203 (2011). Note, that the value of $v$ given there refers to a tennis ball, which is ok for order-of-magnitude purposes
3. TET14 The Engineering Toolbox, http://www.engineeringtoolbox.com/specific-heat-capacity-d_391.html