

# Convection heat transfer between a horizontal surface and the natural environment

D.G. Kröger<sup>1</sup>

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*The problem of heat transfer between an infinite horizontal surface and the natural environment is analysed. When such a surface is heated, due to solar radiation, heat will be transferred to the environment by radiation and convection. The approximate convection heat transfer coefficient under these conditions is determined theoretically and compared to previous experimentally measured values.*

## NOMENCLATURE

$a$	Constant or exponent
$b$	Constant or exponent
$C$	Constant
$C_f$	Friction coefficient
$c$	Constant or exponent
$c_p$	Specific heat, J/kgK
$d$	Exponent
$e$	Exponent
$f$	Exponent
$g$	Gravitational acceleration, m/s <sup>2</sup>
$h$	Heat transfer coefficient, W/mK
$I_h$	Solar irradiation, W/m <sup>2</sup>
$k$	Thermal conductivity, W/mK
$L$	Length, m
$p_a$	Atmospheric pressure, N/m <sup>2</sup>
$T$	Temperature °C or K
$t$	Time, s
$v_w$	Wind speed, m/s
$Z$	Coordinate

## Subscripts

$a$	Air or ambient
$g$	Ground
$i$	Initial condition
$nc$	Natural convection
$o$	At $z = 0$
$oq$	Condition at $z = 0$ for uniform heat flux
$s$	Surface
$sky$	Sky
$u$	Unstable
$w$	Wind

## INTRODUCTION

Consider the energy balance that is applicable to a unit area of horizontal surface that is exposed to the natural environment on a clear, dry, sunny day, as shown in Fig. 1, i.e.

$$I_h \alpha_s = h(T_s - T_a) + \varepsilon_s \sigma (T_s^4 - T_{sky}^4) - k_g (dT/dz) \quad (1)$$

$I_h$  is the incident solar radiation per unit horizontal area.

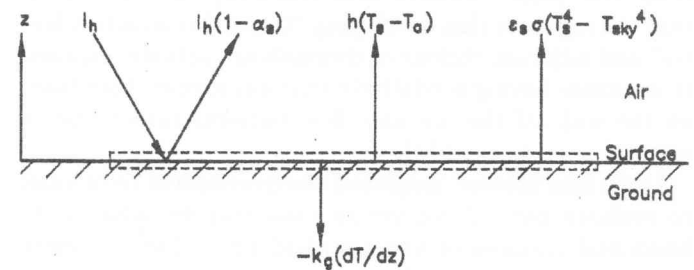


Fig. 1. Heat fluxes at ground surface exposed to environment

The first term on the right-hand side of equation (1) represents the convective heat transfer between the surface and the ambient air. The objective in this study was to determine the turbulent heat transfer coefficient  $h$ .

The second term on the right-hand side of equation (1) represents the longwave radiation between the surface and the environment. In this term, the Kelvin sky temperature can (according to Swinbank<sup>1</sup>) be approximated by  $T_{sky} = 0.0552 T_a^{1.5}$ .

The third term on the right-hand side of equation (1) represents the heat that is conducted into the surface or ground.

## Dimensionless numbers

Gr	Grashof number, $\rho^2 g (T_s - T_a) L^3 / (T \mu^2)$
Nu	Nusselt number, $hL/k$
Pr	Prandtl number, $\mu c_p / k$
Ra	Rayleigh number = $GrPr$ $= \rho^2 g (T_s - T_a) L^3 c_p / (T \mu k)$

## Greek Letters

$\alpha$	Absorptivity or thermal diffusivity, $(k/\rho c_p)$
$\delta$	Conduction layer thickness
$\varepsilon$	Emissivity
$\mu$	Dynamic viscosity, kg/ms
$\rho$	Density, kg/m <sup>3</sup>

1. Department of Mechanical Engineering, University of Stellenbosch, Private Bag X1, Matieland, 7602 South Africa

Many laboratory experiments have been conducted to determine the heat transfer coefficient due to turbulent natural convection from a heated horizontal upward-facing surface. In general the data are correlated by the following relationship between the Nusselt number and the Rayleigh number.<sup>2</sup>

$$Nu = a Ra^{1/3} \quad (2)$$

For air exposed to the heated surface this equation can also be written as

$$h_{nc} [\mu T / \{g(T_s - T_a) c_p k^2 \rho^2\}]^{1/3} = a \quad (3)$$

The properties of the air are evaluated at the mean air temperature  $T = (T_s + T_a)/2$ . Note that the dimensionless number "a" does not contain a length dimension. Equation (3) is applicable to infinite surfaces and is also a good approximation in the case of surfaces having finite dimensions where edge effects are not too significant.

Fujii and Imura<sup>3</sup> refer to the fact that their horizontal test surface or plate was uniformly heated resulting in a maximum temperature at the centre of the plate. To determine the heat transfer coefficient, they employed the average plate temperature. This would give a larger heat transfer coefficient than if they had based their calculations on the centre temperature due to edge effects. Their results also suggest that edge effects play a more significant role in the case of a smaller surface (50 mm) resulting in a larger coefficient ( $a = 0.16$ ) than in the case of a larger surface (300 mm) giving a smaller coefficient ( $a = 0.13$ ).

It is interesting to note that Rohsenow *et al.*<sup>4</sup> refer to the results obtained by Fujii and Imura<sup>3</sup> for upward facing heated surfaces as a situation of "uniform heat flux". Although the heat is introduced uniformly, the 10 mm thick brass test plate conducts heat relatively effectively such that the region of thin developing "thermal conduction layers" and adjacent regions of thermals are actually exposed to a surface having a relatively uniform temperature (seen on the scale of the unstable flow pattern) rather than a uniform heat flux condition.

Lloyd and Moran<sup>5</sup> employed electrochemical techniques to evaluate natural convection mass transfer adjacent to horizontal surfaces of various planforms. The boundary condition of these experiments was uniform concentration at the test surface which is the counterpart of uniform surface temperature in the corresponding heat transfer problem. The dimensions of their rectangular planforms are up to 127 mm × 51.8 mm (of the same order as the smaller surface tested by Fujii and Imura) where edge effects are to be expected. Their coefficient of  $a = 0.15$  should be compared with the value of  $a = 0.16$  obtained by Fujii and Imura. The slightly smaller value obtained by Lloyd and Moran may be due to the fact that their surface potential (concentration) was constant whilst the potential (temperature) in the case of Fuji and Imura was not uniform across the entire surface tested.

Al-Arabi and El-Riedy<sup>6</sup> refer to the work of Kraus who tested 160 mm × 160 mm to 260 mm × 260 mm heated horizontal surfaces and obtained a coefficient of  $a = 0.137$ ,

and Kamal and Salah who studied a horizontal rectangular plate 504 mm × 200 mm maintained at constant temperature and concluded that for a plate of infinite size (for which case the edge effects could be neglected) the value of the coefficient was  $a = 0.135$ . Al-Arabi and El-Riedy<sup>6</sup> carried out experiments on upward facing heated plates at constant temperature. They tested square plates having dimensions varying from 50 mm to 450 mm, circular plates ranging from 100 mm to 500 mm in diameter, and rectangular plates of 150 mm wide and lengths of 250 mm to 600 mm. All their mean results are well correlated by a coefficient of  $a = 0.155$ . They also conducted an experiment on a square plate to find the heat transfer coefficient in the central part of the plate which was not influenced by edge effects. The resultant coefficient had a value of  $a = 0.145$ .

Clausing and Berton<sup>7</sup> investigated the influence of variable properties on the heat transfer due to natural convection from a relatively large (600 mm × 1200 mm) upward facing heated plate. For cases where properties do not change significantly they obtain a coefficient of  $a = 0.14$ .

According to the above investigations, it would appear that for an infinite heated upward-facing horizontal surface that is maintained at a constant temperature the coefficient "a" has some value between 0.13 and 0.16.

Available data for the case of constant heat flux are limited. Lombaard and Kröger<sup>8</sup> conducted experiments on an insulated 1 m × 1 m horizontal plate exposed to the natural environment. They obtained a value of  $a = 0.227$ .

Tests have been conducted on heated surfaces that are exposed to the natural environment during windy conditions. In general the convective heat transfer coefficient for these tests is expressed as

$$h = b + cv_w \quad (4)$$

where  $b$  and  $c$  are supposed to be constants.

It is obvious that this simple equation cannot adequately express the heat transfer coefficient. When the wind speed  $v_w = 0$ , heat is transferred due to natural convection with a value of  $h$  as given by equation (3). Furthermore, equation (4) is not dimensionless and does therefore not make provision for changes in thermo-physical properties.

The result of one of the earliest studies of forced convective heat transfer, conducted by Jurgens who heated a 0.5 m × 0.5 m copper plate mounted vertically and flush with the side of a windtunnel, was reported by Duffie and Beckman,<sup>9</sup> as follows:

$$h = 5.7 + 3.8v_w, \quad W/m^2K \quad (5)$$

Watmuff *et al.*<sup>10</sup> suggest that Jurgens's equations may include free convection and radiation effects overestimating the value of  $h$ . They recommend the following relation

$$h = 2.8 + 3v_w, \quad W/m^2K \quad (6)$$

One of the studies that investigated wind-induced heat transfer from surfaces exposed to real wind was performed

by Test *et al.*<sup>11</sup> A heated plate measuring 1.22 m × 0.81 m was placed in the natural environment, and the wind speed at a height of 1 m above the plate was recorded. They found that

$$h = 8.55 + 2.56v_w, \text{ W/m}^2\text{K} \quad (7)$$

Sharples and Charlesworth<sup>12</sup> obtain results for a surface mounted on a pitched roof and found the following relations for two wind directions (0° and 90°):

$$h = 8.3 + 2.2v_w, \text{ (0°)} \quad (8a)$$

$$h = 6.5 + 3.3v_w, \text{ (90°)} \quad (8b)$$

Recent test results reported by Lombaard and Kröger<sup>8</sup> for a 1 m × 1 m surface located 1.5 m above ground level give  $1.9 \leq c \leq 2.9$ .

The value of "c" appears to lie in the range of 1.9 to 3.3 for tests conducted on relatively small surfaces exposed to the natural environment.

It is not surprising that in view of the above uncertainty, Duffie and Beckman,<sup>9</sup> referring to convective heat transfer due to wind over the surface of a solar collector, state that "from the preceding discussion it is apparent that the calculation of wind induced heat transfer coefficients is not well established". More recently Sharples and Charlesworth<sup>12</sup> came to the conclusion that "... as experiments come closer to resembling "real" collector situations, so more discrepancies and inconsistencies are found both between measured results from different experiments and with standard flat-plate forced convection relationships for  $h$ ."

#### ANALYSIS

In the following analysis an approximate equation is deduced for the convective heat transfer coefficient on a relatively smooth horizontal surface exposed to the environment.

Consider the semi-infinite solid as shown in Fig. 2(a) maintained at some initial temperature  $T_i$ . The surface temperature is suddenly increased and maintained at a temperature  $T_o$ .

To find the temperature distribution as a function of time  $t$  for constant material properties, the following differential equation has to be solved.

$$k \frac{\partial^2 T}{\partial z^2} = \rho c_p \frac{\partial T}{\partial t} \quad (9)$$

According to Schneider<sup>13</sup> the solution to this equation gives a temperature gradient at  $z = 0$  of

$$\partial T / \partial z = (T_i - T_o) / (\pi \alpha t)^{1/2} \quad (10)$$

where  $\alpha = k / (\rho c_p)$ .

The corresponding surface heat flux is

$$q_r = -\frac{k \partial T}{\partial z} = \frac{k(T_o - T_i)}{(\pi \alpha t)^{1/2}} \quad (11)$$

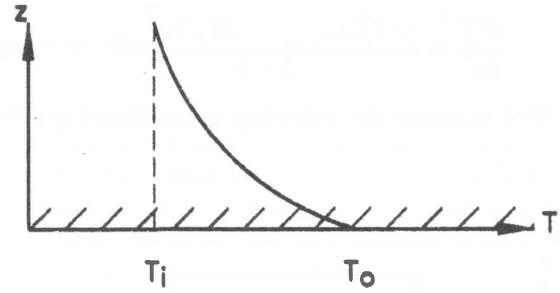


Fig. 2. (a) Nomenclature for transient temperature distribution in a semi-infinite solid

An effective heat transfer coefficient can be expressed in terms of this heat flux, i.e.

$$h_{Tt} = q_T / (T_o - T_i) = k / (\pi \alpha t)^{1/2} \quad (12)$$

Similarly, by solving equation (9) for the case where the semi-infinite solid at a uniform initial temperature  $T_i$  is suddenly exposed to a constant surface heat flux  $q_q$ , the latter can, according to Holman,<sup>14</sup> be expressed in terms of an effective surface temperature  $T_{oq}$  as

$$q_q = k(T_{oq} - T_i) / [2(\alpha t / \pi)^{1/2}] \quad (13)$$

The corresponding effective heat transfer coefficient is defined as

$$h_{qt} = q_q / (T_{oq} - T_i) = k / [2(\alpha t / \pi)^{1/2}] \quad (14)$$

It follows from equations (12) and (14) that for the same temperature difference, i.e for  $(T_{oq} - T_i) = (T_o - T_i)$

$$h_{qt} / h_{Tt} = \pi / 2 = h_q / h_T \quad (15)$$

Equation (9) is also applicable in the region of the early developing temperature distribution of a semi-infinite fluid on a horizontal surface. If the fluid is initially at a uniform temperature  $T_i$  and the surface temperature is suddenly increased and maintained at a temperature  $T_o$ , equations (10) to (12) are applicable. Equations (13) and (14) would apply in the case of constant surface heat flux.

If the Rayleigh number  $Ra \geq 1101$ ,<sup>15</sup> unstable conditions prevail, with the result that the heated fluid is transported upwards away from the surface by means of thermals as shown in Fig. 3. The generation of such thermals is observed to be periodic in time, and, both the spatial frequency and the rate of production are found to increase with an increase in heating rate.

For an analysis of the initial developing temperature distribution near the suddenly heated surface, exposed to a semi-infinite fluid, consider Fig. 2(b).

The approximate magnitude of the curvature of the temperature profile is the same as the change in the slope

$\partial T/\partial z$  across the relatively small conduction layer thickness or height  $\delta$ :

$$\frac{\partial^2 T}{\partial z^2} \approx \frac{(\partial T/\partial z)_{z \sim \delta} - (\partial T/\partial z)_{z \approx 0}}{\delta - 0} \quad (16)$$

Fig. 2(b) suggests the following temperature gradient scales

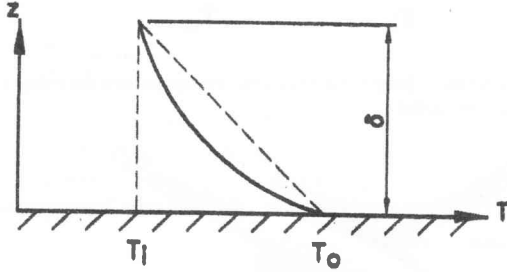


Fig. 2. (b) Early temperature distribution in a semi-infinite fluid

$$(\partial T/\partial z)_{z \approx \delta} = 0, \quad (\partial T/\partial z)_{z \approx 0} \approx (T_i - T_o)/\delta$$

Substitute these gradients into equation (16) and find

$$\partial^2 T/\partial z^2 \approx -(T_i - T_o)/\delta^2 \quad (17)$$

The right-hand side of equation (9) refers to the thermal inertia effect of the conducting material. The approximate magnitude of that term can be deduced by arguing that the average temperature of the  $\delta$ -thick region increases from the initial level  $T_i$  by a value of  $(T_o - T_i)/2$  during the time interval of length  $t$ .

$$\partial T/\partial t \approx -(T_o - T_i)/(2t) \quad (18)$$

According to equations (9), (17), and (18) it may be established that

$$-(T_i - T_o)/\delta^2 \approx -(T_o - T_i)/(2\alpha t)$$

or

$$\delta \approx (2\alpha t)^{1/2} \quad (19)$$

The conduction layer becomes unstable when

$$Ra = g\delta_u^3 (T_o - T_i) \rho^2 c_p / (k\mu T) = 1101 \quad (20)$$

where  $T = (T_o + T_i)/2$  in K.

At this condition

$$\delta_u = 10.33 [k\mu T / \{g(T_o - T_i) \rho^2 c_p\}]^{1/3} \quad (21)$$

From equations (19) and (21) it follows that

$$t_u \approx 53.31 [\mu T / \{g(T_o - T_i)\}]^{2/3} (c_p/k\rho)^{1/3} \quad (22)$$

Substitution of equation (22) into equation (12) leads to

$$h_{Tt} [\mu T / \{g(T_o - T_i) c_p k^2 \rho^2\}]^{1/3} = 0.0773$$

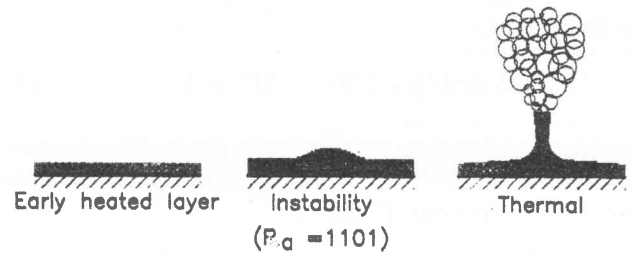


Fig. 3. Thermal-flow development in vicinity of heated surface

The average heat transfer coefficient during the period  $t$  is found by integrating equation (12) resulting in

$$h_T = 2k/(\pi\alpha t_u)^{1/2} = 2h_{Tt}$$

or

$$h_T [\mu T / \{g(T_o - T_i) c_p k^2 \rho^2\}]^{1/3} = 0.155 \quad (23)$$

If the surface is suddenly exposed to a constant heat flux it follows from equation (15) that  $h_q = \pi h_T/2$  or

$$h_q [\mu T / \{g(T_{oq} - T_i) c_p k^2 \rho^2\}]^{1/3} = 0.243 \quad (24)$$

It is stressed that these equations are only applicable to the first phase of the heat transfer process and do not include the second phase, during which thermals exist. No simple analytical approach is possible during this latter phase although the mean heat transfer coefficient during the breakdown of the conduction layer will probably not differ that much from the first phase.

The agreement between this approximate and only partial analysis with experimentally measured values at constant surface temperatures i.e.  $a = 0.155$  (according to experiments  $0.13 \leq a \leq 0.16$ ) is satisfactory. For the case of constant heat flux  $a = 0.243$  which compares with a value of  $a = 0.227$  obtained experimentally by Lombaard and Kröger.<sup>8</sup>

Equation (24) would be applicable to most surfaces that have a relatively low thermal conductivity (most natural surfaces) and are exposed to solar radiation.

During windy periods the complex nature of turbulent flow in the atmosphere is referred to by Blackader.<sup>16</sup> Bejan<sup>17</sup> notes that the viscous boundary sublayer emerges as a laminar shear layer that develops (grows) until it becomes unstable and breaks down at a local Reynolds number, based on the local thickness of order 100. The breakdown of the laminar layer is punctuated by a burst in which the slow fluid is ejected into the faster moving turbulent core, as shown schematically in Fig. 4. Since the viscous sublayer breaks down periodically due to the local Reynolds number becoming supercritical, the viscous sublayer maintains effectively the same thickness (time averaged superposition of laminar shear layers) regardless of the overall thickness of the boundary layer.

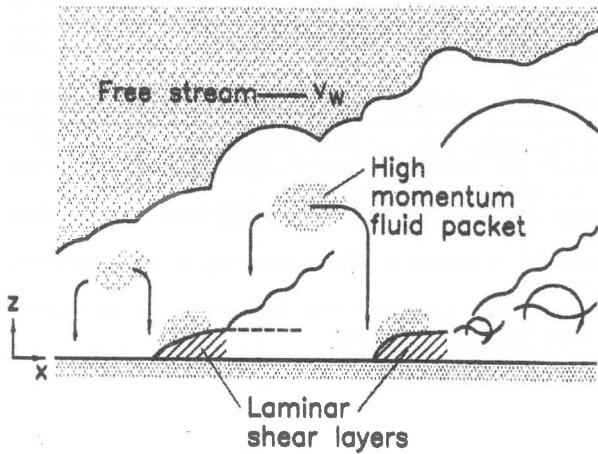


Fig. 4. Formation of laminar shear layers

To find the heat transfer coefficient during windy periods, the analogy between the boundary layer air flow (momentum transfer) and heat transfer will be relied upon.

According to the Reynolds-Colburn analogy, the convective heat transfer coefficient due to the wind can be expressed as<sup>17</sup>

$$h_w = C_f \rho c_p v_w / \left[ 2 (\mu c_p / k)^{2/3} \right] \quad (25)$$

Since the viscous sublayer effectively maintains the same thickness, the corresponding effective skin friction coefficient  $C_f$  will be essentially constant which means that the heat transfer coefficient  $h_w$  will be directly proportional to the wind speed  $v_w$ .

The applicability of equation (25) is confirmed by dimensional analysis. During windy periods the heat transfer coefficient may be a function of buoyancy  $g(\rho_i - \rho_o) = g\Delta\rho$  and the other independent parameters applicable to natural convection as well as the wind speed  $v_w$ , i.e.

$$h_w = f(g\Delta\rho, \mu, k, \rho, c_p, v_w)$$

or dimensionally

$$\frac{kg}{Ks^3} = f \left[ \left( \frac{kg}{m^2s^2} \right)^a \left( \frac{kg}{ms} \right)^b \left( \frac{kgm}{Ks^3} \right)^c \right. \\ \left. \times \left( \frac{kg}{m^3} \right)^d \left( \frac{m^2}{Ks^2} \right)^e \left( \frac{m}{s} \right)^f \right]$$

such that

$$kg : 1 = a + b + c + d$$

$$m : 0 = -2a - b + c - 3d + 2e + f$$

$$s : -3 = -2a - b - 3c - 2e - f$$

$$K : -1 = -c - e$$

With  $h_w$  proportional to  $v_w$ , i.e.  $f = 1$ , find the exponents  $a = 0, b = e - 1, c = 1 - e$  and  $d = 1$  such that

$$h_w = C_f \rho c_p v_w / (\mu c_p / k)^{(1-e)} \quad (26)$$

This equation is the same as equation (25) if the constant  $C = C_f/2$  and  $(1 - e) = 2/3$  or  $e = 1/3$ .

According to available experimental data, the value of "c" in equation (4) is in the range of 1.9 to 3.3, i.e. the average value is  $c = 2.6$  for finite surface areas having different geometries and orientations. From equations (4) and (25) it follows that

$$C_f \rho c_p v_w / \left[ 2 (\mu c_p / k)^{2/3} \right] = c v_w$$

$C_f$  in the case of an infinite area may differ considerably from the above-named range of values. Further experimental work is required to evaluate the influence of variations in wind speed, direction, etc. on the values of  $C_f$ .

Combining equations (23) and (26) leads to the following expression for the dimensionless convective heat transfer coefficient, valid for a surface at constant temperature, exposed to the natural environment.

$$h_T [\mu T / \{g(T_o - T_i) c_p k^2 \rho^2\}]^{1/3} \quad (27)$$

$$= 0.155 + v_w (C_f/2) [\rho T / \{\mu g(T_o - T_i)\}]^{1/3}$$

Similarly for a surface subject to a uniform heat flux

$$h_q [\mu T / \{g(T_{oq} - T_i) c_p k^2 \rho^2\}]^{1/3} \quad (28)$$

$$= 0.243 + v_w (C_f/2) [\rho T / \{\mu g(T_{oq} - T_i)\}]^{1/3}$$

Most natural surfaces tend to have a relatively low thermal conductivity. When exposed to solar radiation the corresponding convective heat transfer coefficient as given by equation (28) will be applicable.

## CONCLUSION

Equations for the convective heat transfer coefficients between an infinite horizontal surface and the natural environment are deduced. These equations highlight the difference in heat transfer coefficient between a surface that is maintained at a constant temperature and one that is exposed to a uniform heat flux. The present approximate analytical model predicts values of heat transfer coefficients for natural convection that are in good agreement with experimentally measured value. Additional experimental data are however required to obtain a more reliable value for the friction coefficient  $C_f$  on an infinite surface during windy periods. The term that contains and predicts the influence of wind is a new dimensionless number. Equation (28) is applicable to most relatively smooth natural surfaces exposed to the environment.

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