Dynamic Portfolio Management with Views at Multiple Horizons

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Agenda

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• The market model
• Portfolio construction
• Case studies
### Background

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<td>Black-Litterman ('90)</td>
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<td>Entropy Pooling ('08)</td>
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- The standard approach to **discretionary portfolio management** (Black-Litterman, Entropy Pooling) processes subjective views that refer to the distribution of the market at a specific single investment horizon.
- The standard approach to **multi-period portfolio management with market impact** (Garleanu-Pedersen) processes **non-discretionary** (systematic) signals.
- Dynamic Entropy Pooling is a quantitative approach to perform dynamic portfolio management with **discretionary, multi-horizon views**.
The profit-and-loss (P&L)

- We assume the single-period P&L is a set of exposures multiplied by the increments of the risk drivers over the rebalancing period:

\[ \Pi_{(t,t+1]} = b'_t (X_{t+1} - X_t) \]

- The set of risk drivers can be extended to include also external factors that do not affect directly the P&L of the instruments. On such additional factors we can express views that influence the P&L through correlation. The corresponding entries in the exposures vector will be set to zero.
Equities

- We assume the single-period P&L is a set of exposures multiplied by the increments of the risk drivers over the rebalancing period:

\[
\Pi_{(t,t+1)} = b'_t (X_{t+1} - X_t)
\]

- The set of risk drivers can be extended to include also external factors that do not affect directly the P&L of the instruments. On such additional factors we can express views that influence the P&L through correlation. The corresponding entries in the exposures vector will be set to zero.

- Consider an equity share or an index. Then the risk driver is its log-value:

\[
X_t = \ln V_t
\]

- The P&L of a portfolio with \( h_{n,t} \) shares in the \( n \)-th asset is:

\[
\Pi_{(t,t+1)} = \sum_n h_{n,t} V_{n,t} \times \left( \frac{V_{n,t+1}}{V_{n,t}} - 1 \right) \approx \sum_n b_{n,t} \Delta X_{n,t+1}
\]

- More in general, in terms of a style/risk linear factor model:

\[
\Pi_{(t,t+1)} = \sum_k b_{k,t}^{style} \Delta X_{k,t+1}^{style}
\]
Fixed-Income

- We assume the single-period P&L is a set of exposures multiplied by the increments of the risk drivers over the rebalancing period:

$$\Pi(t, t+1) = b_t'(X_{t+1} - X_t)$$

- The set of risk drivers can be extended to include also external factors that do not affect directly the P&L of the instruments. On such additional factors we can express views that influence the P&L through correlation. The corresponding entries in the exposures vector will be set to zero.

- Suppose that the $n$-th asset is a fixed income instrument. Its value at the first order satisfies

$$\Pi_{n,(t,t+1)] \approx -\sum_k dv01_{n,k,t} \Delta X_{k,t+1}$$

where $X_{k,t}$ is the $k$-th key-rate on the yield curve; $dv01_{n,k,t}$ is the dollar-sensitivity of the $n$-th instrument to $X_{k,t}$.

- Then the P&L due to a set of fixed income instruments is:

$$\Pi(t, t+1) \approx \sum_k \left( -\sum_n h_{n,t} dv01_{n,k,t} \right) \Delta X_{k,t+1}$$
Options

- We assume the single-period P&L is a set of exposures multiplied by the increments of the risk drivers over the rebalancing period:

\[ \Pi_{(t,t+1]} = b_t'(X_{t+1} - X_t) \]

- The set of risk drivers can be extended to include also external factors that do not affect directly the P&L of the instruments. On such additional factors we can express views that influence the P&L through correlation. The corresponding entries in the exposures vector will be set to zero.

- For a stock option, the risk drivers are the log-value of the underlying \( X = \ln V \) and the implied volatility \( \Sigma^{impl} \).

- Then for a portfolio of stock options, the P&L is:

\[ \Pi_{(t,t+1]} \approx \sum_n h_n,t \delta_{n,t} V_n,t \Delta X_{n,t+1} + \sum_n h_n,t v_{n,t} \Delta \Sigma_{n,t+1}^{impl} \]

where \( \delta_{n,t} \) and \( v_{n,t} \) are the delta and vega of the \( n \)-th option.
Multivariate Ornstein-Uhlenbeck (MVOU) process

- Consider a book of assets driven by a set of risk drivers $X_t$ (interest rates, implied volatility surfaces, log-prices, etc.)
- We assume that the drivers follow a MVOU process:

$$dX_t = (-\theta X_t + \mu)dt + \sigma dW_t$$

- Choose a set of discrete monitoring dates $t, t + 1, \ldots, \bar{t}$
- Stack the process at the monitoring times as follows:

$$X_{t \sim \bar{t}} = \begin{pmatrix} X_t \\ X_{t+1} \\ \vdots \\ X_{\bar{t}} \end{pmatrix}$$

- Then the process is jointly multivariate normal at all times

$$X_{t \sim \bar{t}} | \iota_t \sim N(\mu_{t \sim \bar{t}}, \sigma_{t \sim \bar{t}}^2)$$
MVOU expectation and covariance

- The vector of the expectations is

$$\mu_{t \sim \tilde{t}} = \begin{pmatrix} e^{-0\theta} x_t + \left( I_n - e^{-0\theta} \right) \theta^{-1} \mu \\ e^{-1\theta} x_t + \left( I_n - e^{-1\theta} \right) \theta^{-1} \mu \\ e^{-(\tilde{t}-t)\theta} x_t + \left( I_n - e^{-(\tilde{t}-t)\theta} \right) \theta^{-1} \mu \end{pmatrix}$$

- The covariance matrix is

$$\sigma^2_{t \sim \tilde{t}} = \begin{pmatrix} \sigma_0^2 & \sigma_0^2 e^{-\theta'} & \sigma_0^2 e^{-2\theta'} \\ e^{-\theta} \sigma_0^2 & \sigma_1^2 & \sigma_1^2 e^{-\theta'} \\ e^{-2\theta} \sigma_0^2 & e^{-\theta} \sigma_1^2 & \sigma_2^2 \end{pmatrix}$$

where

$$vec(\sigma^2_{\tau}) \equiv (\theta \oplus \theta)^{-1} \left( I_{n^2} - e^{-(\theta \oplus \theta)\tau} \right) vec(\sigma^2)$$
Posterior distribution from general prior

We extend the Entropy Pooling approach in Meucci (2010) to the case of multiple horizons

- **The prior**: assume a model for the joint distribution of the process at the monitoring times:
  \[ X_t \sim X_t | i_t \sim f \]

- **The views**: are statements (constraints) on the yet-to-be defined distribution of the process:
  \[ g \in \mathcal{V}_t \]

- **The posterior**: is the closest distribution to the prior that satisfies the views:
  \[ \bar{f} \equiv \arg\min_{g \in \mathcal{V}_t} \mathcal{E}\{(g, f)\} \]

where the “distance” is the relative entropy

\[ \mathcal{E}(g, f) \equiv \int g(x_t, \ldots, x_{\bar{t}}) \ln \frac{g(x_t, \ldots, x_{\bar{t}})}{f(x_t, \ldots, x_{\bar{t}})} dx_t \cdots x_{\bar{t}} \]
Posterior distribution from MVOU prior

We extend the Entropy Pooling approach in Meucci (2010) to the case of multiple horizons

- **The prior**: assume a MVOU model for the joint distribution of the process at the monitoring times

\[
X_{t \sim \bar{t}} | i_t \sim N(\mu_{t \sim \bar{t}}, \sigma^2_{t \sim \bar{t}})
\]

- **The views**: are statements (constraints) on the yet-to-be defined distribution of the process:

\[
\mathcal{V}_t : \begin{align*}
E_t^g \{ v_{\mu,t} X_{t \sim \bar{t}} \} &\equiv \mu_{\text{view};t} \\
C_v g \{ v_{\sigma,t} X_{t \sim \bar{t}} \} &\equiv \sigma^2_{\text{view};t}
\end{align*}
\]

where \(v_{\mu,t}\) and \(v_{\sigma,t}\) are matrices that define arbitrary linear combinations of the process at the times for the views.

- **The posterior**: is the closest distribution to the prior that satisfies the views:

\[
\bar{f}_X \equiv \arg\min_{g \in \mathcal{V}_t} \mathcal{E}\{(g, f)\} \Rightarrow X_{t \sim \bar{t}} | i_t \sim N(\bar{\mu}_{t \sim \bar{t}}, \bar{\sigma}^2_{t \sim \bar{t}})
\]
Posterior distribution from MVOU prior

\[ X_{t \sim \bar{t}} | i_t \sim N(\bar{\mu}_{t \sim \bar{t}}, \bar{\sigma}^2_{t \sim \bar{t}}) \]

- For the expectation, we introduce the pseudo inverse matrix of \( v_{\mu, t} \)

\[ v^+_\mu, t \equiv \sigma^2_{t \sim \bar{t}} v'_\mu, t (v_{\mu, t} \sigma^2_{t \sim \bar{t}} v'_\mu, t)^{-1} \]

and we define the two complementary projectors:

\[ P_{\mu, t} \equiv (I_{n(\bar{t}-t+1)} - v^+_\mu, t v_{\mu, t}), \quad P_{\mu, t}^\perp \equiv v^+_\mu, t v_{\mu, t} \]

Then

\[ \bar{\mu}_{t \sim \bar{t}} \equiv P_{\mu, t} \mu_{t \sim \bar{t}} + P_{\mu, t}^\perp (v^+_{\mu, t} \mu_{\text{view}, t}) \]

- Similar, for the covariance we introduce the pseudo inverse of \( v_{\sigma, t} \)

\[ v^+_\sigma, t \equiv \sigma^2_{t \sim \bar{t}} v'_\sigma, t (v_{\sigma, t} \sigma^2_{t \sim \bar{t}} v'_\sigma, t)^{-1} \]

and the two complementary projectors:

\[ P_{\sigma, t} \equiv I_{n(\bar{t}-t+1)} - v^+_\sigma, t v_{\sigma, t}, \quad P_{\sigma, t}^\perp \equiv v^+_\sigma, t v_{\sigma, t} \]

Then

\[ \bar{\sigma}^2_{t \sim \bar{t}} \equiv P_{\sigma, t} \sigma^2_{t \sim \bar{t}} P_{\sigma, t}^\perp + P_{\sigma, t}^\perp (v^+_{\sigma, t} \sigma^2_{\text{view}, t} (v^+_{\sigma, t})') (P_{\sigma, t}^\perp)' \]
Objective

- As in Garleanu and Pedersen (2013), the satisfaction functional is an infinite sum of discounted trade-offs:

$$S_t^{(\gamma, \eta)} = \mathbb{E}_t \left\{ \sum_{s=t}^{\infty} e^{-\lambda(s-t)} \left( \mathbb{E}_s \{ \Pi_{s+1} \} - \frac{\gamma}{2} \mathbb{V}_s \{ \Pi_{s+1} \} - \frac{\eta}{2} \mathbb{E}_s \{ MI_s \} \right) \right\}$$

where the market impact is a quadratic function of the exposure rebalancing

$$MI_t = a^2 + (b_t - b_{t-})' c^2 (b_t - b_{t-}) = a^2 + \Delta b_t' c^2 \Delta b_t$$

with $c^2$ a suitable positive definite matrix. Note the term $a^2$, which represents the average cost of maintaining constant exposures.
Objective as function of exposures

- Given that the P&L is linear in the exposures $\Pi_{t+1} = b'_t \Delta X_{t+1}$, we need to solve for the optimal policy of exposures as functions of information

\[
\{b^*_s = p^*_s(I_s)\}_{s \geq t}, \text{ where}
\]

\[
\{p^*_s\}_{s \geq t} = \arg\max_{\{p_s\}_{s \geq t} \in \mathcal{C}} E_t \left\{ \sum_{s=t}^{\infty} e^{-\lambda(s-t)} [p_s(I_s)' \omega \bar{E}_s \{\Delta X_{s+1}\} - \frac{\gamma}{2} p_s(I_s)' \omega \mathcal{V}_s \{\Delta X_{s+1}\} \omega' p_s(I_s) - \frac{\eta}{2} \Delta p_s(I_s)' c^2 \Delta p_s(I_s)] \right\}
\]

- As in Garleanu and Pedersen (2013), the satisfaction functional is an infinite sum of discounted trade-offs:

\[
\bar{S}^{(\gamma, \eta)}_t \equiv E_t \left\{ \sum_{s=t}^{\infty} e^{-\lambda(s-t)} (\bar{E}_s \{\Pi_{s+1}\} - \frac{\gamma}{2} \mathcal{V}_s \{\Pi_{s+1}\} - \frac{\eta}{2} \bar{E}_s \{MI_s\}) \right\}
\]

where the market impact is a quadratic function of the exposure rebalancing

\[
MI_t = a^2 + (b_t - b_{t-})' c^2 (b_t - b_{t-}) = a^2 + \Delta b'_t c^2 \Delta b_t
\]

with $c^2$ a suitable positive definite matrix. Note the term $a^2$, which represents the average cost of maintaining constant exposures
General solution

- Given that the P&L is linear in the exposures $\Pi_{t+1} = b'_t \Delta X_{t+1}$, we need to solve for the optimal policy of exposures as functions of information.

  \[
  \{b^*_s = p^*_s(i_s)\}_{s \geq t}, \text{ where } \\
  \{p^*_s\}_{s \geq t} = \arg\max_{p_s} \mathbb{E}_t \{ \sum_{s=t}^{\infty} e^{-\lambda(s-t)} [p_s(l_s)' \omega \mathbb{E}_s \{ \Delta X_{s+1} \} \\
  - \frac{\gamma}{2} p_s(l_s)' \omega \mathbb{C} v_s \{ \Delta X_{s+1} \} \omega' p_s(l_s) - \frac{\eta}{2} \Delta p_s(l_s)' c^2 \Delta p_s(l_s)] \} 
  \]

- Dynamic programming with a quadratic value function yields a recursive problem with time-dependent coefficients

  \[
  v_{s+1}(b_s, x_{s+1}) = -\frac{1}{2} b'_s \psi_{bb,s} b_s + b'_s \psi_{bx,s} x_{s+1} + \frac{1}{2} x'_{s+1} \psi_{xx,s} x_{s+1} + \psi'_{b,s} b_s + \psi'_{x,s} x_{s+1} + \psi_{0,s}
  \]

  \[
  \Leftrightarrow \psi_{s-1} = g_s(\psi_s)
  \]

- The optimal policy of exposures then reads

  \[
  b^*_s = q_s^{-1} \left[ \eta c^2 b_{s-1} + (\omega \beta_s + e^{-\lambda} \psi_{bx,s}(\beta_s + \Pi_{\beta})) x_s + (\omega + e^{-\lambda} \psi_{bx,s}) x_{s+1} + \psi_{b,s} \right]
  \]

  where $q_s = \gamma \omega \bar{\sigma}_s^2 \omega' + \eta c^2 + e^{-\lambda} \psi_{bb,s}$
Special case: no market impact

- Given that the P&L is linear in the exposures $\Pi_{t+1} = b_t' \Delta X_{t+1}$, we need to solve for the optimal policy of exposures as functions of information

$$\{b_s^* = p_s^*(i_s)\}_{s \geq t}, \text{ where }$$

$$\{p_s^*\}_{s \geq t} = \arg\max_{\{p_s\}_{s \geq t} \in C} E_t\left\{ \sum_{s=t}^{\infty} e^{-\lambda(s-t)} [p_s(I_s)'] \omega \overline{E}_s \{\Delta X_{s+1}\} - \frac{\gamma}{2} p_s(I_s)' \omega \mathbb{C} V_s \{\Delta X_{s+1}\} \omega' p_s(I_s) - \frac{\eta}{2} \Delta p_s(I_s)' c^2 \Delta p_s(I_s) \right\}$$

- With no market impact, we obtain a series of myopic one-period problems
- The optimal policy is a sequence of mean-variance optimizations based on the posterior distribution of the risk drivers process

$$b_s^* = \frac{1}{\gamma} (\omega \overline{\sigma}_s^2 \omega')^{-1} \omega (I_{\mu,s})_{s+1} \cdot \Delta \mu^\text{LongTerm}_{s \sim t} \left\arrow \begin{pmatrix} I_n - e^{-\theta} \\ I_n - e^{-1 \theta} \\ I_n - e^{-(\bar{t} - s) \theta} \end{pmatrix} (\theta^{-1} \mu - x_s)$$

$$b_s^\text{LongTerm} = \frac{1}{\gamma} (\omega \overline{\sigma}_s^2 \omega')^{-1} \omega (I_{\perp \mu,s})_{s+1} \cdot \Delta \mu^\text{ViewMean}_{s \sim \tilde{t}} \left\arrow (v_{\mu,s}^{\perp \mu \text{view};s} - x_s)$$

$$b_s^\text{ViewMean} =$$

Dynamic Portfolio Management with Views at Multiple Horizons
A. Meucci, M. Nicolosi
One risk driver, one view

\[ b_t^{Prior} \]

\[ b_t^{LongTerm} \]

\[ b_t^{viewMean} \]
Two risk drivers (one investable), two views

\( b_{t}^{LongTerm,X_1} \)

\( b_{t}^{LongTerm,X_2} \)

\( b_{t}^{ViewMean,X_1} \)

\( b_{t}^{ViewMean,X_2} \)

Comparison of \( b_{t}^{*} \)
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