Mathematical Ideas that Emerge from Data Collected in the Classroom

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Activity	Mathematics Involved
The Hand Squeeze	Linear functions
That's the Way the Ball Bounces	Linear functions
Total Stopping Distance	Quadratic functions
Jump!	Quadratic functions
Paper Folding – Mythbusters	Exponential functions (growth)
Value of Car	Exponential functions (decay)
Modeling the Spread of a Rumor	Logistic Growth
Breaking Paperclips	Power functions
Hershey's Kiss	Quartic functions – volume of solid of revolution

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AMATYC 2007 – Minneapolis, MN

The Hand Squeeze

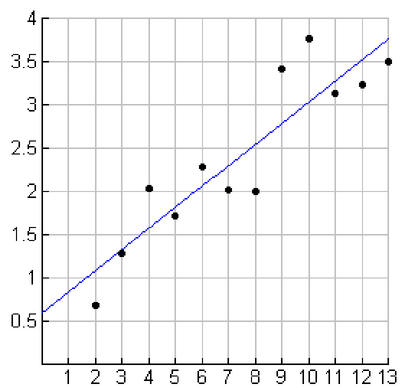
Set Up

Start with two people standing face-to-face holding hands (like at a wedding). A third student can be the time keeper. When the time keeper says go, the designated squeeze starter squeezes one of the hands of the second person. Upon feeling the squeeze, this second person squeezes back with the other hand. The squeeze starter shouts “Stop!” when the squeeze is felt and the time keeper stops the stopwatch. Record the time. Continue this procedure by adding one person to the circle each iteration.

Sample Data

The following data were collected in an Intermediate Algebra class in the Spring of 2004.

People	2	3	4	5	6	7	8	9	10	11	12	13
Time (seconds)	0.69	1.29	2.04	1.72	2.29	2.02	2.00	3.41	3.76	3.13	3.24	3.50



Linear Regression (ax+b)
regEQ(x) = .243462x + .598205
r = .903828
r^2 = .816905

Suggested Questions

1. Determine a linear function that models the data.
2. Describe the meaning of the constant rate of change in this situation.
3. Describe the meaning of the vertical intercept in this situation.
4. Using the linear model, determine the time it would take for the squeeze to travel from beginning to end if there were 30 people involved.
5. If the time keeper recorded a time of 10 seconds for the squeeze to travel from beginning to end, how many people were involved according to the linear model you found?
6. Explain the meaning of the coefficient of determination.
7. Explain the meaning of the correlation coefficient.
8. Since a linear model was used in this situation, we would say that the rate of change is constant. Explain what constant rate of change means in the context of this situation.
9. Suppose each person was precisely 0.20 seconds slower than what was recorded. How could you adjust the linear model found in #1 and write the equation of the linear function that models this new data set?
10. Compare the slope of the linear function found in #1 with the slope of the linear function in #9. What do you notice? Why is this the case?

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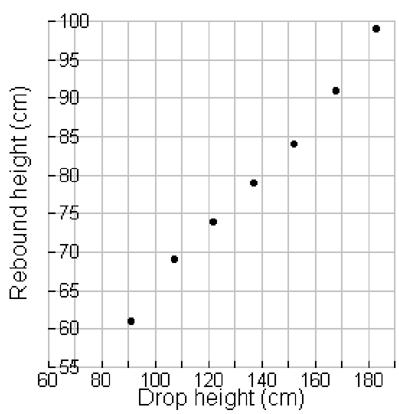
That's the Way the Ball Bounces

Set Up

In this activity, students measure the rebound height of different types of balls when they are dropped from different initial heights. Students will tape cloth tape measures to the wall and drop the ball from several different heights, carefully measuring the rebound height. It is often helpful to collect at least 3 rebound heights for each initial height, taking the average rebound height as the data point. It is interesting to give each group a different type of ball so that comparisons can be made. Golf balls, racquetball balls, tennis balls, ping pong balls, etc. all work well.

Sample Data – Ping Pong Ball

Initial Height (cm)	91	107	122	137	152	168	183
Rebound Height (cm)	61	69	74	79	84	91	99



Linear Regression (ax+b)
regEQ(x) = .392707x + 25.7144
a = .392707
b = 25.7144
r^2 = .993423

Suggested Questions

1. Determine a linear function that models the data.
2. Describe the meaning of the constant rate of change in this situation.
3. Describe the meaning of the vertical intercept in this situation.
4. Using the linear model, determine the rebound height for a ball dropped from an initial height of 200 cm.
5. If the rebound height recorded was 55 cm, what was the initial height from which the ball was dropped?
6. Explain the meaning of the coefficient of determination.
7. Explain the meaning of the correlation coefficient.
8. Since a linear model was used in this situation, we would say that the rate of change is constant. Explain what constant rate of change means in the context of this situation.
9. Suppose rebound height was precisely 2.5 cm less than what was recorded. How could you adjust the linear model found in #1 and write the equation of the linear function that models this new data set?
10. Compare the slope of the linear function found in #1 with the slope of the linear function in #9. What do you notice? Why is this the case?

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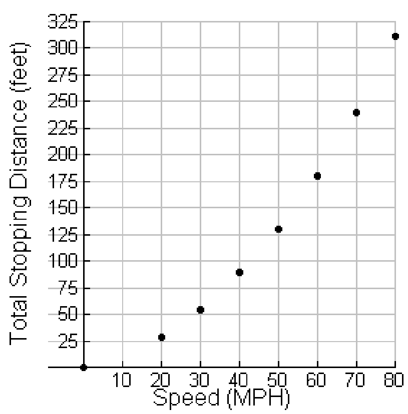
Total Stopping Distance

Set Up

In this activity, students measure the stopping distance of a car traveling different speeds by using an applet found at www.phy.ntnu.edu.tw/ntnujava/viewtopic.php?t=224.

Sample Data

Speed (MPH)	0	20	30	40	50	60	70	80
Stopping Distance (feet)	0	28.61	54.83	88.88	129.76	180.72	239.91	311.63



Quadratic Regression
regEQ(x) = .042051x^2 + .494397x + .958725
a = .042051
b = .494397
c = .958725
R^2 = .999834

Suggested Questions

1. Determine a quadratic function that models the data.
2. Describe the meaning of the each of the parameters, *a*, *b*, and *c* in this situation..
3. Describe the meaning of the vertical intercept in this situation.
4. Using the quadratic model, determine the total stopping distance for a car traveling 100 MPH.
5. It is discovered that the total stopping distance of a car was 200 feet. How fast was the car traveling before braking?
6. Explain the meaning of the coefficient of determination.
7. Estimate the average rate of change on [0,50]. What does this value represent in this situation?
8. Use the quadratic model to estimate the average rate of change on [50, 55]. What does this value represent in this situation?
9. Since a quadratic model was used in this situation, we would say that the rate of change of the rate of change is constant. Explain what the rate of change of the rage of change means in the context of this situation.
10. Suppose stopping distance was precisely 50 feet less (except for the first data point) than what was recorded. How could you adjust the quadratic model found in #1 and write the equation of the quadratic function that models this new data set?

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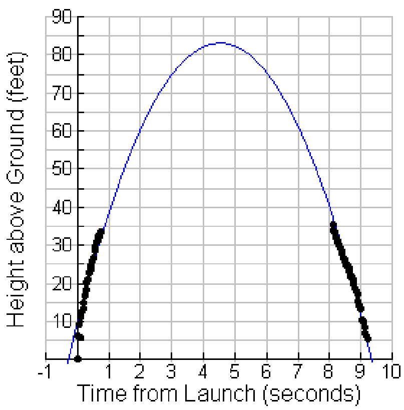
Jump

Set Up

In this activity, students measure the height above the ground of a car relative to time based on a television commercial for a Dodge vehicle. While the video is computer generated, it still provides an interesting context by which to investigate quadratic functions.

Sample Data



Quadratic Regression
regEQ(x) = -3.58316x^2 + 32.4858x
a = -3.58316
b = 32.4858
c = 9.4179
R^2 = .937268

Time	Height
0	0
0.01	5.777338
0.03	6.295817
0.07	9.110417
0.10	11.18433
0.13	12.73077
0.17	13.33232
0.20	15.40358
0.23	16.8876
0.27	18.22083
0.30	20.22068
0.34	20.96137
0.37	23.03528
0.40	23.77597
0.44	24.88699
0.47	25.77581
0.50	26.81277
0.54	28.66448
0.57	29.77551
0.60	30.7384
0.64	31.33095
0.67	32.3679
0.70	32.51604
0.74	33.33079
8.11	35.40471
8.14	33.92334
8.18	33.33079
8.21	31.84942
8.24	31.40501
8.28	30.81247
8.31	29.99771
8.34	29.40517
8.38	28.66448
8.41	27.77566
8.44	26.81277
8.48	26.44243
8.51	25.77581
8.54	24.81292
8.58	24.81292
8.61	23.85003
8.64	23.25749
8.68	21.92426
8.71	21.77612
8.74	20.8873
8.78	19.92441
8.81	18.59118
8.84	18.0727
8.88	17.10981
8.91	15.4803
8.94	14.29521
8.98	13.33232
9.01	13.33232
9.04	10.44365
9.08	10.14737
9.11	8.517869
9.14	6.814296
9.18	6.073671
9.21	5.629204
9.24	5.406995

Suggested Questions

1. Determine a quadratic function that models the data.
2. Describe the meaning of the each of the parameters, *a*, *b*, and *c* in this situation..
3. Determine the maximum height above the ground attained by the car according to the model. At what time was this height attained?
4. Using the quadratic model, determine the height of the car after 5 seconds.
5. At what time(s) was the height of the car 70 feet above the ground?
6. Explain the meaning of the coefficient of determination.
7. Estimate the average rate of change on [0,1]. What does this value represent in this situation?
8. Use the quadratic model to estimate the average rate of change on [8, 9]. What does this value represent in this situation?
9. Since a quadratic model was used in this situation, we would say that the rate of change of the rate of change is constant. Explain what the rate of change of the rage of change means in the context of this situation.
10. Use the quadratic model to determine the total amount of time that the car was in the air.

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Paper Folding (Mythbusters)

Set Up

To begin, ask students to fold a piece of paper in half as many times as they can. They should keep track of the number of times they fold the paper. Then, ask them to measure the thickness of the paper. Next, pose the hypothetical situation where we are able to fold a piece of paper 40 times. How thick will the wad of paper be after 40 folds? Most students think linearly rather than exponentially and underestimate greatly.

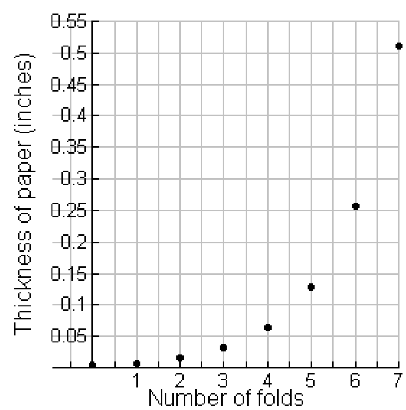
The Mythbusters show dealt with this situation by investigating if it is possible to fold a piece of paper more than 7 times. This myth was explored in episode 72 (http://dsc.discovery.com/fansites/mythbusters/episode/episode_02.html).

Sample Data

According to http://hypertextbook.com/facts/2001/JuliaSherlis.shtml, copy paper is 0.004 inches thick.

Number of Folds	0	1	2	3	4	5	6	7
Paper Thickness (inches)	0.004	0.008	0.016	0.032	0.064	0.128	0.256	0.512

Suggested Questions



1. Determine an exponential function that models the data.
2. Describe the meaning of the growth factor in the context of this situation.
3. Write an equivalent exponential function where the growth factor is 4.
4. Using the exponential model, determine the paper thickness after 40 folds.
5. If the paper thickness was about 524 feet, how many times was the paper folded?
6. Can you find the inverse of this function? How can you know if you are able? If you are able, determine the inverse function.
7. We can say that the thickness of the paper increases by 100% after each fold. Explain how this relates to a growth factor of 2.
8. We can say that the exponential model in this case has a constant growth factor of 2. How is this different from a linear situation where we would find a constant rate of change?
9. Using the language of rate of change, describe the behavior of this function as the number of folds increases from 0 to 10.
10. A student says that the function to model this paper folding situation is $f(x) = 2x + 0.004$. How is this student thinking about the situation and what might you do or say to correct this student's thinking?

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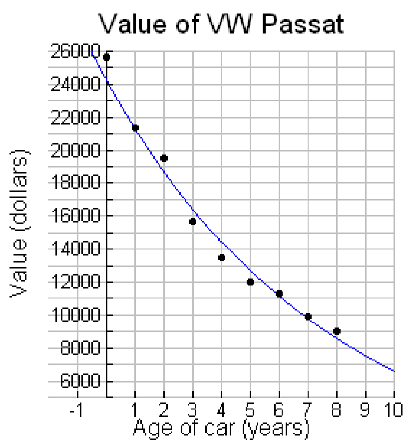
Exponential Decay

Set Up

To simulate the decreasing value of an automobile, use the Kelly Blue book website, www.kbb.com, to find the value of a new car and the values of older models of the same car. For example, a new Volkswagen Passat (2008) is valued at $25,630. This is seen in the table as a car of age 0. The car with age of 1 is the Kelly Blue Book value of a 2007 Passat. The data given was found at www.kbb.com in October 2007.

Sample Data

Age of Car (years)	0	1	2	3	4	5	6	7	8
Value of Car (dollars)	25630	21375	19535	15645	13500	12000	11260	9910	9065



Exponential Regression
regEQ(x) = 24351.9*.877648^x
r = -.991317
r^2 = .98271

Suggested Questions

1. Determine an exponential function that models the data.
2. Describe the meaning of the growth factor in this situation.
3. Describe the meaning of the vertical intercept in this situation.
4. Using the exponential model, determine the value of the 2008 Passat after 15 years.
5. At what age will he Passat have a value of $6000?
6. Explain the meaning of the coefficient of determination.
7. Explain the meaning of the correlation coefficient.
8. We can say that the value of the Passat decreases by about 13% each year. Explain how this relates to a growth factor of 0.87.
9. We can say that the exponential model in this case has a constant growth factor of approximately 0.87. How is this different from a linear situation where we would find a constant rate of change?
10. Using the language of rate of change, describe the behavior of this function as time increases from 0 to 10.

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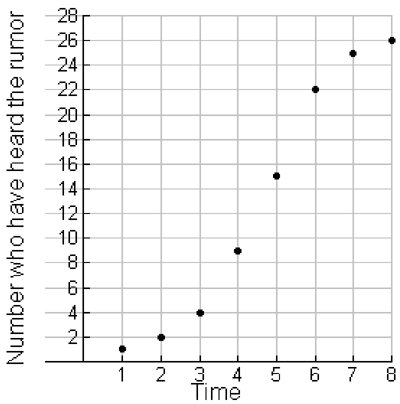
Spread of a Rumor

Set Up

To simulate the spread of a rumor, use your graphing calculator to generate random numbers. Ask the students to count off from 1 to *n* and then generate random numbers to indicate when the student has heard the rumor. Generate one random number to determine who begins the rumor. Then generate another random number to represent who the rumor starter told the rumor to first. Next, generate 2 random numbers to represent the 2 students who were told next. Continue to generate random numbers equal to the number of students who have heard the rumor at that time. As time goes on, some students will have already heard the rumor – do not count them again. Continue until all students have heard the rumor. The site http://education.ti.com/educationportal/activityexchange/Activity.do?cid=US&aId=7709 has a similar activity related to the television show Numb3rs.

Sample Data

Time	1	2	3	4	5	6	7
Number of People	1	4	9	15	22	25	26



Logistic Regression
regEQ(x) = 27.3168/(1+116.932*#e^(-1.00859x))

Suggested Questions

1. Determine a logistic function that models the data.
2. Using the language of rate of change, describe the behavior of this function as time increases from 0 to 8.
3. Determine the point of inflection and explain its significance in this situation.
4. Describe the significance of the value of the numerator of the logistic model.
5. How does the rate at which people hear the rumor change over time?
6. Explain why the rate at which people become infected slows down towards the end of the experiment.
7. The graph appears to approach a horizontal line as time increases. What does this horizontal line represent? How can you use your logistics model to find the equation of this horizontal line?
8. Compute the average rate of change on [0, 3]. Explain what this value represents.
9. Compute the average rate of change on [4, 8]. Explain what this value represents.
10. Using the language of rate of change, describe the behavior of this function as time increases from 0 to 8.

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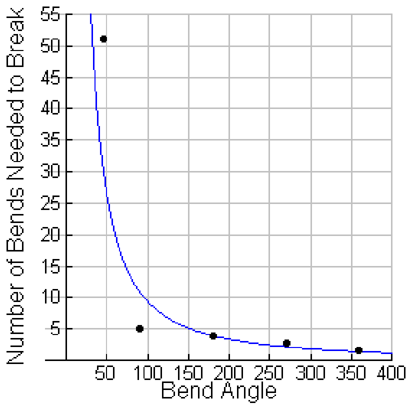
Paperclips

Set Up

Perform a stress test on each paperclip by selecting an angle at which you can repeatedly bend the paper clip until it breaks. Repeat the stress test on each of the paper clips, using a different angle for each one.

Sample Data

Angle	45	90	180	270	360
Number of Bends Before Breaking	51	5	4	2.66666	1.5



Power Regression
regEQ(x) = 9076.03x^-1.49108
r^2 = .872583

Suggested Questions

1. Explain what should happen, in the context of the situation, to the dependent variable as the independent variable approaches 0.
2. Explain what should happen, in the context of the situation, to the dependent variable as the independent variable approaches infinity.
3. Based on your answers to #1 and #2, explain why a power regression model might best represent this situation. Determine the power function that models this situation.
4. Using the power function model, determine the number of bends before breaking a paper clip under normal conditions. Assume that normal conditions involve a 5 degree angle.
5. If the paper clip broke after 30 bends, at what angle was the paper clip being bent?
6. Explain the meaning of the coefficient of determination.
7. What is the practical domain for the power function model?
8. What is the practical range for the power function model?
9. Use the language of rate of change to describe the behavior of the function on (0, 360].
10. Compute the average rate of change on [10, 360]. Explain what this value means in the context of the situation.

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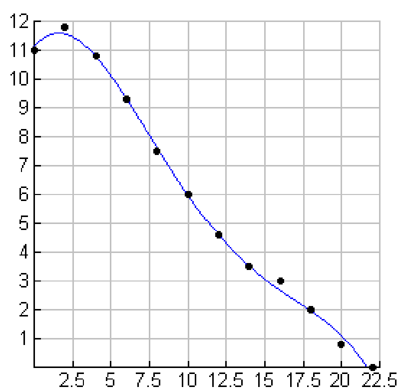
How Much Chocolate is in that Little Hershey's Kiss?

Set Up

- Put the Kiss on the graph paper and create a region bounded by the outline of the Kiss and the x-axis such that when the region is revolved around the x-axis, the entire Kiss is formed.
- Find 12 points on the outline of the Kiss. Record them and label them on the graph.
- Enter the points into the calculator lists such that all x values are in L_1 and y values in L_2.
- Determine the regression model for the outline of the Kiss.
- Set up an integral which would determine the volume of the solid form if the region is revolved around the *x*-axis.
- Use your calculator or Maple to evaluate the definite integral.
- Determine the units used in this experiment and convert them to cubic centimeters, if necessary. How much chocolate is in that little Hershey's Kiss?
- Determine the amount of foil needed to wrap the Kiss. In what units is your answer?
- Compare your answers with other groups.

Sample Data

x (mm)	0	2	4	6	8	10	12	14	16	18	20	22
y (mm)	11	11.8	10.8	9.3	7.5	6	4.6	3.5	3	2	0.8	0



Quartic Regression
regEQ(x) = -.000298x^4 + .014679x^3 + -.230683x^2 + .624785x + 11.124
R^2 = .99809

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2 mm by 2mm Grid

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