Strengthening Mathematics Courses for Elementary Education

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Abstract

Explore ways to incorporate abstract mathematics concepts into courses for pre-service elementary teachers. Engage in hands-on activities to solve problems from the student's perspective. See projects and other activities useful for the classroom. Also hear about a possible math minor for elementary education majors.

1 Introduction

After Mount Royal started offering Bachelor of Education in Elementary, I led the creation of Minor in Mathematics for Elementary Education [16]. This program gives the students in the Bachelor of Education, Elementary to have the option to increase their enthusiasm for and some expertise in the field of Mathematics. In this program, the student is required to take the three core courses and then choose three courses with at least one course must be 3000 level or above from the approved Mathematics options in order to attain a total of six courses of which at least two courses must be 3000 level or above. Furthermore, I Spearheaded the development of curriculum and the design of four new courses: Mathematical Reasoning, Higher Arithmetic, Mathematics through Ages, and Seminar for this minor. The first two courses with a primary focus on elementary school mathematics from an advanced viewpoint.

Because we have the luxury of teaching in an environment where our professor to student ratio is quite high, we have been able to take innovative steps to improve student learning.

As stated in [10], when teaching mathematics, teaching math courses for future teachers differs from teaching traditional courses such as calculus and linear algebra. Also, the Conference Board of Mathematical Sciences (CBMS) argues that mathematics content courses should not only aim to remedy weaknesses in mathematical knowledge, but also help teachers develop a deeper and more comprehensive view and understanding of the mathematics they will or already do teach. ([12], p. 23). Further, [9] stated that "Mathematics and statistics departments have the responsibility to ensure that future mathematics teachers have deep and connected understandings of the mathematics they will teach. " In [11] author analyzed 20 textbooks and concluded that " lessons from the books by mathematicians are that the mathematics of elementary school has deep and complex roots; that there are different and sometimes conflicting approaches to explaining this mathematics; and that there may be no perfect mathematical solution to the problem of how to teach this subject." To this end, I have created inquiry-based problem-solving activities which lead to introduction of abstract concepts, and also I have created lecture notes as an open educational resources to facilitate student learning.

2 Core courses

2.1 Mathematical reasoning

The purpose of this course is to introduce discrete mathematics concepts, with a primary focus on elementary school mathematics from an advanced viewpoint.The relationship of concepts to the elementary mathematics curriculum was emphasized. The lecture notes for this course can be found in [13]. These lecture notes are created as an open educational resource.

2.2 Higher Arithmetic

This course explores elementary number theory, numeration systems, operations on integers and rational number and elementary combinatorics using both inductive and deductive methods. Emphasis will be put on the development of clarity and understanding of mathematical processes and ideas, the application of these ideas to problem solving and the communication of these ideas to other people. The lecture notes for this course can be found in [14]. These lecture notes are created as an open educational resource.

2.3 Seminar

Students present seminars and discuss topics in mathematics taken from current journals or books. Instruction and practice in written and oral communication is provided. The topics vary from student to student.

3 Mathematics through ages-Optional course

Students have done this course as a directed reading course and followed a multidisciplinary learning approach. A large portion of the subject focused on the history of mathematics. I have facilitated the learning throughout the course, exploring patterns of mathematical practice in different historical contexts: Greek, Middle Ages, Scientific Revolution, Age of Enlightenment, and the twentieth century. We were intrigued by the diversity of such cultures throughout the history of mathematics. This was the beginning of the research project - an opportunity to gain deep cultural understanding of teaching and learning mathematics, [15].

This paper analyzes the range and focus of cultural influences on teaching practices, operating in different parts of the world. It is hoped that this research will shed light on the role of culture as an important aspect of mathematics teaching and learning. The report also provides information that may be useful in the effective design of teaching pedagogies. It will also be of use to teachers' both pre-service or experienced - as they consider how to invest in or evaluate mathematical teaching practices.

4 Activities

Here are some of the activities that I have done in class before introducing the connected abstract concepts. These activities are introduced as "Thinking Out Loud:". In this talk I will engage the participants with the some of the following hands on activities.

4.1 Visual Proof & Sand paper art

The goal of this activity to visualize number pattern using sand paper art. The materials needed for this activity areSand paper, acrylic paint, sticks

4.2 Fibonacci exploration

In this activity we will explore the following problem, A tiling consists of covering a region using tile pieces from some given set so that the region is completely covered without overlaps. How many ways can you arrange the 2×1 dominoes to cover $2 \times n$ checker board.

4.3 More Fibonacci

Let a_n be the Fibonacci sequence. That is $a_n = a_{n-1} + a_{n-2}, n \in \mathbb{N},$ with $a_1 = 1, a_0 = 0$, where $\mathbb{N}$ be the set of all natural numbers. Consider the Fibonacci squares illustrated in figure.

Let h_n be the shortest (perpendicular) distance between n^{th} parallel diagonal vectors. Consider the figure:

Comparing the the area of trapezoids, we get

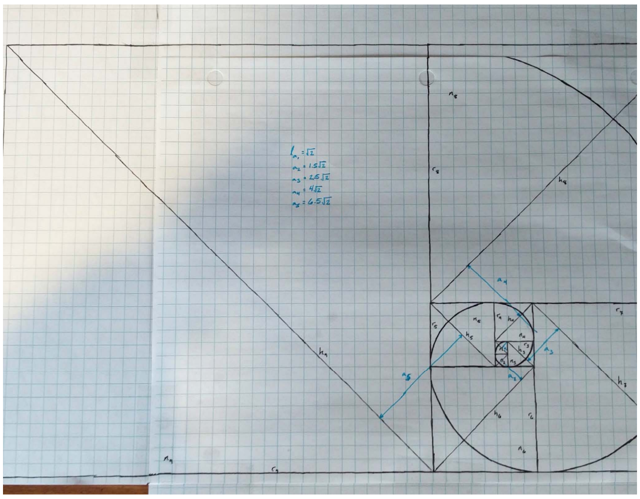
$$h_n = \frac{a_n^2 - a_{n-1}^2}{\sqrt{2}(a_n - a_{n-1})} = \frac{1}{\sqrt{2}} \left(a_n + a_{n-1}\right).$$

Hence, h_n is fibonacci.

4.4 Holiday decorations

The participants will construct holiday decorations (really the Platonic solids). A large variety of geometric topics can be built into the lesson and/or discussed afterword. Any platonic solid can be made, but I'll stick to the tetrahedrons. The tetrahedron makes nice holiday decoration.

Figure 1: Fibonacci like sequence



4.5 Platonic solids with magnaformers

The goal of this activity is to appreciate polygons and support the idea that there are exactly 5 platonic solids. My students and I have created a video for this activity, as a part of an open educational resource.

4.6 Paper folding

Show that if you take the midpoints of any quadrilateral and connect them in turn, you will always get a parallelogram.

5 Conclusion

It appears that students learned and understood the concepts better. Moreover, they enjoyed working on the group activities and discussions in class.

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Strengthening Mathematics Courses for Elementary Education by **Pamini Thangarajah**

Platonic solids with Magnaformers

Goal: To appreciate polygons and support the idea that there are exactly 5 platonic solids.

Terminology:

- A polygon is a closed 2-dimensional figure with straight sides
  - An n-gon is a polygon with exactly n sides
  - A regular n-gon is a polygon with exactly n sides, where all sides are of equal length and all interior angles of the polygon are equal. The sum of the interior angles of a regular n-gon is 180°(n - 2). It follows that each interior angle must measure 180°(n - 2)/n. So:
    - A regular 3-gon is an equilateral triangle. Each interior angle is 60°
    - A regular 4-gon is a square. Each interior angle is 90°
    - A regular 5-gon is a regular pentagon. Each interior angle is 108°
    - A regular 6-gon is a regular hexagon. Each interior angle is 120°
    - A regular 7-gon is a regular heptagon. Each interior angle is 900/7°, or approximately 128.6°
    - A regular 8-gon is a regular octagon. Each interior angle is 135°

Activity:
Suppose I want to tape regular n-gons together to make 3-dimensional shapes. I can make a cube, for example, by taping squares together. What are my options? I don't want to bend or fold the n-gons. Let's just concentrate on the corners of these objects.

Fact: To make a corner I'll need at least 3 regular n-gons.
Try making corners out of 3 n-gons. Which ones will work? Justify your conclusions.

Now try using four n-gons to make corners. Which ones will work? Justify your conclusions.
What about using five n-gons? Justify your conclusions.
Can we make corners out of six or more n-gons? Justify your conclusions.

A platonic solid is a 3-dimensional object made by taping together regular n-gons in such a way that each corner is the same, and has the same number of n-gons around it. Using the data you've gathered, please complete the following statement:

I have found that there are ____________ ways to tape regular n-gons together to make the corners of a platonic solid. Therefore, there are at most __________ platonic solids.

Holiday Decorations (as an excuse to problem solve in geometry!)

Grade level: This activity can be tailored to various grade levels

Outcomes: Students will construct holiday decorations (really the Platonic solids). A large variety of geometric topics can be built into the lesson and/or discussed afterword. Any platonic solid can be made but I'll stick to the cube and icosahedrons. The cube because it is geometrically the simplest and makes nice gift boxes and an icosahedron because I think it is visually the most striking.

Motivational strategies:

- The resulting craft is very attractive and students will see its construction as an enjoyable and worthy endeavor.
- The teacher gets to choose the appropriate amount of mathematics to put into the activity and where to put the mathematics in. You can measure angles and lengths, discuss properties of circles and the regular polygons, apply the Pythagorean theorem to build a cube of a particular size, use the law of cosines or sine law to build an icosahedron of a particular size, discuss surface area and of course talk about the Platonic solids and other polyhedra.

Materials/Equipment:

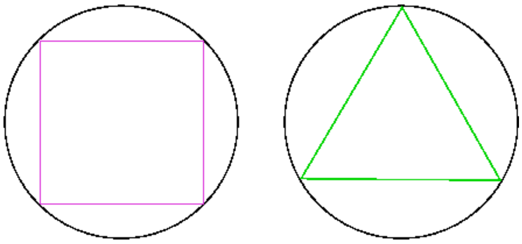
- A circle cutter (borrow from a crafty person or buy for 20-50 dollars) -get a metric one if you can!
- Card stock, Bristol board, used greeting cards or some other stiff but not too thick paper. You'll need lots of it for a class.
- White glue.

Process: You might want to look at the websites in the references to begin to see potential final versions of the decorations and make one or two on your own to get a feeling for time involved and difficulty. They are really just the Platonic solids with or without flaps. Unlike the website descriptions however, we'll use a circle cutter to speed the activity along. On the next page I will outline the directions for making the decorations. After that I will discuss the potential geometry you can discuss before, after or during the construction of the decorations.

CONSTRUCTION DIRECTIONS:

Step 1: Fix a radius on your circle cutter and mass produce a lot of circles. For example, if students were making cubes they'd need 6 per cube, if they make the icosahedrons they'll need 20 circles (in my opinion icosahedrons with flaps are the nicest to make!) Keeping the radius fairly small (eg 4 inches on my non-metric circle cutter) would let you get away with using up less paper and makes a nice sized decoration. Produce some extra circles for errors and a geometry discussion with students.

Step 2: Take one of the circles and inscribe the template square (for cubes) or equilateral triangle (for icosahedrons). (Fold to get the center of the circle and for the square the folds locate the vertices for the square. For the triangle you can pull out a protractor and measure off 3 120 degree angles to get the vertices.) Cut out the template shape.



Step 3: Trace this shape and trace it out on as many circles as you need. Work fast – slight imperfections are ok. Now take a rule and a nail (or something else a little sharp) and quickly trace over the lines to break the surface of the cardstock. Fold the sides up to get little 'cups' with a square or triangular base. I have gone through steps one to three and made over 120 circles folded into cups in one hour.

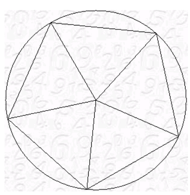
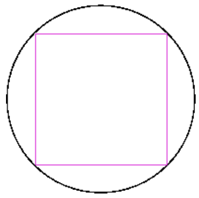
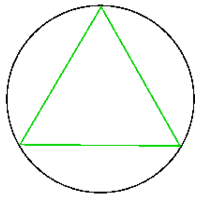
Step 4: Bring nets of the cube and icosahedrons (or pictures from the websites mentioned) to help guide the students in putting the shape together. Students will be gluing the flaps together to form the cube or icosahedrons. You have the options of gluing the flaps out or flaps in. The icosahedrons looks really neat with the flaps out. Given them the white glue and lead them through putting it together. Again, make one yourself in advance so you can get a feel for how much guidance your students will need and if they need an older helper to finish it off.

<u>Some ideas for the geometry:</u>

1. Before you begin, show students a finished decoration and how it is actually constructed from squares/ equilateral triangles inscribed in circles. Show them a circle cutter and discuss how it works (a compass with a blade is much cooler than a compass with a pencil in it!)

2. Give them a blank circle and have them problem solve how to find its center and inscribe a square in it. (Folding will do it). If they know about angles and protractors,

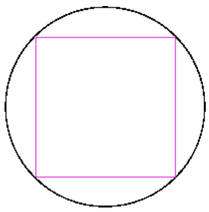
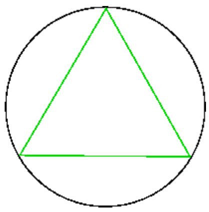
ask them how to inscribe an equilateral triangle into it and ask why it works. You have the option (if appropriate for your grade level) to go into the general idea of inscribing regular polygons into circles. The regular n sided polygon can be thought of as made up of n identical triangles each triangle having two sides of length the radius of the circle and angle (360/n) degrees. (See picture below) From here you can get to the formula for the interior angles of a regular polygon.

3. For lower grades you can simply ask them to check the angles are equal (and what they are) and check that the side lengths are the same to confirm we have an equilateral triangle/square.

4. Ask: I want to make a cube with side length X what radius should I set the circle cutter at to get the cube? You need the Pythagorean Theorem to answer that one! Warning: If your circle cutter is not in metric it is a little ugly. (8.3 inches is harder to explain than 8.3 cm) (See below)

5. Ask: I want to make an icosahedron with side length X, what radius should I set the circle cutter at to get the equilateral triangles. You need the Law of Cosines or Law of Sines to get that one! (see below)

6. You can talk about area and surface area. How much paper was wasted? What is the total surface area of the resulting cube/icosahedrons. (One could always find the area of an equilateral triangle by measuring).

7. And of course, you can talk about the Platonic solids as well! In fact, you can also construct other polyhedra (the Archimedean solids for example) using this technique. I'd need the Pythagorean theorem to construct a cuboctahedron.