Developing Writing Skills of Calculus Students

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Frustration correcting repetitive wrong math notations

- Correcting exactly same error over and over and over again.
- Complaints for deducting 1 point or 0.5 points for wrong usage of math notation.

The Fact is…

It is too late to correct students' writing error by the time you collect 1st test.

It is very crucial to correct students' writing errors at early stage of the semester!

Grading

- Grading is the least favorite part of my job, BUT this is the job AND ONE MUST DO THEIR JOB.
- Grading is the most essential element to know the students.
- Grading is the indispensable element to assess my own students throughout the semester.

Calculus Students

- They are going to transfer to the university and continue taking further math courses.
- It is important they are writing correct usage of math notations in the subsequent courses in order to communicate with their professors.

Danger of wrong math usage

- Wrong math usage gives an impression that the person has no idea about what he or she is talking about even if they may know what they are talking about.
- Wrong math usage leads misunderstanding.

Common Writing Errors

$$\lim_{n \to \infty} = 0$$

On the corner of the test paper…

$\sin x = cosx$

$\cos x = -sinx$

$\tan x = sec^2x$

$\csc x = -csccotx$

$\sec x = secxtanx$

$\cot x = -csc^2x$

This is worse…

sin= cos

cos= $-sin$

tan= sec^2x

csc= $-csccot$

sec= $secxtanx$

cot= $-csc^2x$

$$ln x = \frac{1}{x}$$

$$(ln x)' = \frac{1}{x}$$

$$\int_{1}^{3} (x^3 + x)dx$$

$$= \int \frac{x^4}{4} + \frac{x^2}{2} \Big]_1^3 dx$$

$$\frac{(1) - (-2)}{(1) - (-2)}$$

$$\frac{f(1) - f(-2)}{(1) - (-2)}$$

$$1 = 2$$

$$f(1) = 2$$

Idea from Prof. Baney CC of Baltimore County

Homework's the Beginning: Moving Beyond Mechanics in Calculus II

at AMATYC Conference in New Orleans 2015

Incorporation of Webassign Homework assignments (about 10-15 questions each section) and Worksheets.

3 worksheets examples
by Prof. Baney

Grading Big Papers

Central Oregon Community College

Regular Calculus lectures plus Lab activity.

Cutting Lab Activities into small portions to adapt the class activity at each class

3 Labs examples

Grading on Group Projects

Group Project

- Only motivated student do them all.
- Other student gets all helps from motivated student and gets free credits by doing nothing.

Worksheet at each class

- 1-2 pages in order to introduce main concept at each class.
- Go over exactly same questions in class.
- Students clean up their writings on worksheets within 1 week after the worksheet is covered in class and turn in the paper.
- Graded and corrected any writing errors by the next class period.

Grade in MTH 174 (Calculus II)

- Test 1 18% (Area and Volume, Integration by Parts, Trig Integration)
- Test 2 18% (Rest of the techniques of integrations)
- Test 3 18% (Infinite Series)
- Final 26% (Test 1, 2, and 3, plus Differential Equations, Parametric Equations, Arc Length Surface Area, etc.)
- Web-assign Homework 10%
- Worksheets 10%

MTH 174 at Thomas Nelson

Stewart Calculus Early Transcendental 8th Edition

Order of the course

- Chapter 6 (Volume of rotated objects)
- Chapter 7 (Techniques of integrations)
- Chapter 11 (Sequences and Series)
- Chapter 8 (Arc Length, Surface Area of rotated objects)
- Chapter 9 (Differential Equations)
- Chapter 10 (Polar Coordinates, Parametric Equations)

-.5 for each writing error

- Not all students correct their mistake from the next time.
- Some students will never change the way they write no matter what.
- Majority of the students understand that is an error and become aware of how they write what they write from the 2nd paper.

Grading each worksheet for 24 students

30 minutes to 1 hour for each worksheet .

1st Test

Students are very aware of deduction when they make a writing error.

VERY smooth grading, since few corrections on writing errors.

Same grading criteria as worksheet (-.5 for each writing error)

6.2 : Practice MEASUREMENTS for volume problems

For each problem:

1. Graph the area. Make the area big enough to see easily.
2. Draw the appropriate representative rectangle.
3. **3.** List the measurements of your shape, using exactly the lists of measurements.
4. **STOP! Don't do anything else.**

(1) area bounded by: $y^2 = x$, $y = \frac{1}{2}x$.
rotate around: the x-axis.

The shape generated by the rectangle will be a...

measurements:

thickness=

start=

end=

(2) area bounded by: $y^2 = x$, $y = \frac{1}{2}x$.
rotate around: the y-axis.

The shape generated by the rectangle will be a...

measurements:

thickness=

start=

end=

(3) area bounded by: $y^2 = x$, $y = \frac{1}{2}x$.
rotate around: the line $y = 2$.

The shape generated by the rectangle will be a...

measurements:

thickness=

start=

end=

(4) area bounded by: $y^2 = x$, $y = \frac{1}{2}x$.
rotate around: the line $x = -1$.

<u>The shape generated by the rectangle will be a...</u>

<u>measurements:</u>

thickness=

start=

end=

(5) area bounded by: $y^2 = x$, $y = \frac{1}{2}x$.
rotate around: the line $y = -3$.

<u>The shape generated by the rectangle will be a...</u>

<u>measurements:</u>

thickness=

start=

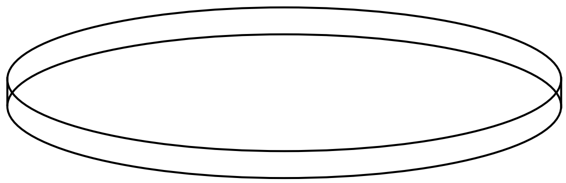
end=

Summary: Formulas and Measurements for Volume Problems

Disks and Washers:

Representative rectangle perpendicular to the axis of rotation.

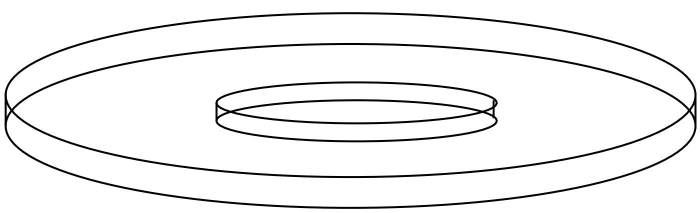
Disk: rep. rect. touches axis of rotation.



$$Volume = \pi \cdot (radius)^2 \cdot thickness$$

measurements:
- *thickness* = width of the representative rectangle
  - *start* = where you start counting rectangles
  - *end* = where you stop counting rectangles
- *radius* = from the axis of rotation to the outside edge of the representative rectangle

Washer: rep. rect. does not touch axis of rotation.



$$Volume = \pi \cdot (R^2 - r^2) \cdot thickness$$

measurements:
- *thickness* = width of the representative rectangle
  - *start* = where you start counting rectangles
  - *end* = where you stop counting rectangles
- *R* = *outer radius* = from the axis of rotation to the outside edge of the representative rectangle
- *r* = *inner radius* = from the axis of rotation to the inside edge of the representative rectangle

7.1 Practice Integration By Parts

Order of Priority for "u" (to differentiate)
L: log(ln)
I : Inverse Trig
A: Algebraic
T: Trig
E: Exponential

(1) $\int x \cdot \cos(x)dx$	
(2) $\int x \cdot e^{x}dx$	

(3) $\int \ln(x)dx$	
(4) $\int x^2 \cdot \ln(x)dx$	

(5) $\int x^2 \cdot \cos(x)dx$	

11.8 Worksheet

Find the values of x (the interval) for which the series converges. Find a formula for the sum.

(1) $$\sum_{n=1}^{\infty} \frac{x^n}{3^n}$$

(2) $$\sum_{n=1}^{\infty} (x-4)^n$$

(3) $$\sum_{n=0}^{\infty} 4^n x^n$$

(4) $$\sum_{n=0}^{\infty} \frac{(x+3)^n}{2^n}$$

(5) $$\sum_{n=0}^{\infty} \left(\frac{\cos(x)}{2} \right)^n$$

Maclaurin and Taylor Series

Maclaurin and Taylor series are a way to approximate almost all functions (including trig functions, exponentials, logarithms, and rational and radical functions) as power series, which are polynomials with an infinite number of terms -- remember, this is a good thing because polynomials are easier to work with than all those other kinds of functions.

Suppose that f(x) can be approximated by a power series -- can we find the coefficients c_0 , c_1, c_2 , c_3 , ... ?   Yes, here's how:

Let $f(x) = c_0 + c_1x + c_2x^2 + c_3x^3 + c_4x^4 + \cdots$

let x = 0, so $f(0) = c_0 + c_1(0) + c_2(0)^2 + c_3(0)^3 + c_4(0)^4 + \cdots$

$f(0) = c_0$

or $\boxed{c_0 = f(0)}$

Next, take the derivative of f(x):

$f'(x) = c_1 + 2c_2x + 3c_3x^2 + 4c_4x^3 + \cdots$

let x = 0, so $f'(0) = c_1 + 2c_2(0) + 3c_3(0)^2 + 4c_4(0)^3 + \cdots$

$f'(0) = c_1$

or $\boxed{c_1 = f'(0)}$

Next, take the derivative of f'(x):

$f''(x) = 2c_2 + 3 \cdot 2c_3x + 4 \cdot 3c_4x^2 + 5 \cdot 4c_5x^3 + 6 \cdot 5c_6x^4 \cdots$

let x = 0, so $f''(0) = 2c_2 + 3 \cdot 2c_3(0) + 4 \cdot 3c_4(0)^2 + 5 \cdot 4c_5(0)^3 + 6 \cdot 5c_6(0)^4 \cdots$

$f''(0) = 2c_2$

or $\boxed{c_2 = \frac{f''(0)}{2}}$

Next, take the derivative of f''(x):

$f'''(x) = 3 \cdot 2 \cdot 1 \cdot c_3 + 4 \cdot 3 \cdot 2 \cdot c_4x + 5 \cdot 4 \cdot 3 \cdot c_5x^2 + 6 \cdot 5 \cdot 4 \cdot c_6x^3 \cdots$

let x = 0, so $f'''(0) = 3 \cdot 2 \cdot 1c_3 + 4 \cdot 3 \cdot 2c_4(0) + 5 \cdot 4 \cdot 3c_5(0)^2 + 6 \cdot 5 \cdot 4c_6(0)^3 + \cdots$

$f'''(0) = 3 \cdot 2 \cdot 1 \cdot c_3$

or $\boxed{c_3 = \frac{f'''(0)}{3 \cdot 2 \cdot 1}}$

... and so on!

Let's summarize the coefficients we've found, and look for a pattern:

$$c_0 = f(0) \Rightarrow c_0 = \frac{f(0)}{0!}$$ Remember: 0! = 1.

$$c_1 = f'(0) \Rightarrow c_1 = \frac{f'(0)}{1!}$$

$$c_2 = \frac{f''(0)}{2} \Rightarrow c_2 = \frac{f''(0)}{2!}$$

$$c_3 = \frac{f'''(0)}{3 \cdot 2 \cdot 1} \Rightarrow c_3 = \frac{f'''(0)}{3!}$$

and, in general:

$$c_n = \frac{f^{(n)}(0)}{n!}$$

The series we've just found is called the <u>Maclaurin Series</u> -- the infinite polynomial (power series) centered at x = 0, which can approximate almost any function f(x):

The Maclaurin Series

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$$

$$f(x) = f(0) + f'(0) \cdot x + \frac{f''(0)}{2!} x^2 + \frac{f'''(0)}{3!} x^3 + \frac{f^{(4)}(0)}{4!} x^4 + \frac{f^{(5)}(0)}{5!} x^5 + \cdots$$

To help make this seem reasonable, let's look at just the first two terms:

$$f(x) \approx f(0) + f'(0) \cdot x$$

y-intercept: b

slope of function at 0: m

$$\Rightarrow b + mx \Rightarrow mx + b\,!$$

in other words,

$f(x) \approx$ the tangent line at x = 0, which <u>is</u> the best possible linear approximation of f(x) at x = 0. Way cool!

<u>Important Fact</u>: The first two terms of the Maclaurin series give the tangent line to the function at x = 0.

Now, we could go through the exact same analysis to find the coefficients of the Taylor Series -- but the only difference between the Taylor and Maclaurin series is that the Taylor series can be centered at any point x = a.   So:

The Taylor Series

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

$$f(x) = f(a) + f'(a)\cdot(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \frac{f^{(4)}(a)}{4!}(x-a)^4 + \cdots$$

Again, to help make this seem reasonable, let's look at just the first two terms:

$$f(x) \approx f(a) + f'(a)\cdot(x-a)$$

y-value at x = a: y_1

slope of function at a: m

x-value at x = a: x_1

$\Rightarrow y_1 + m(x - x_1) \Rightarrow m(x - x_1) + y_1$

$\Rightarrow$ pt-slope formula of a line!

in other words,

$f(x) \approx$ the tangent line at x = a, which is the

best possible linear approximation of f(x) at x = a.   Still way cool!

Important Fact: The first two terms of the Taylor series give the tangent line to the function at x = a.

Chart for constructing the Maclaurin Series for any function:

$f(x) =$	$f(0) =$	$c_0 = \frac{f(0)}{0!} =$	$c_0 \cdot x^0 =$
$f'(x) =$	$f'(0) =$	$c_1 = \frac{f'(0)}{1!} =$	$c_1 \cdot x^1 =$
$f''(x) =$	$f''(0) =$	$c_2 = \frac{f''(0)}{2!} =$	$c_2 \cdot x^2 =$
$f'''(x) =$	$f'''(0) =$	$c_3 = \frac{f'''(0)}{3!} =$	$c_3 \cdot x^3 =$
$f^{(4)}(x) =$	$f^{(4)}(0) =$	$c_4 = \frac{f^{(4)}(0)}{4!} =$	$c_4 \cdot x^4 =$
$f^{(5)}(x) =$	$f^{(5)}(0) =$	$c_5 = \frac{f^{(5)}(0)}{5!} =$	$c_5 \cdot x^5 =$
...	...	...	...
$f^{(n)}(x) =$	$f^{(n)}(0) =$	$c_n = \frac{f^{(n)}(0)}{n!} =$	$c_n \cdot x^n =$

Then $f(x) = \sum_{n=0}^{\infty} c_n x^n$

Chart for constructing the Taylor Series for any function:

$f(x) =$	$f(a) =$	$c_0 = \frac{f(a)}{0!} =$	$c_0 \cdot (x-a)^0 =$
$f'(x) =$	$f'(a) =$	$c_1 = \frac{f'(a)}{1!} =$	$c_1 \cdot (x-a)^1 =$
$f''(x) =$	$f''(a) =$	$c_2 = \frac{f''(a)}{2!} =$	$c_2 \cdot (x-a)^2 =$
$f'''(x) =$	$f'''(a) =$	$c_3 = \frac{f'''(a)}{3!} =$	$c_3 \cdot (x-a)^3 =$
$f^{(4)}(x) =$	$f^{(4)}(a) =$	$c_4 = \frac{f^{(4)}(a)}{4!} =$	$c_4 \cdot (x-a)^4 =$
$f^{(5)}(x) =$	$f^{(5)}(a) =$	$c_5 = \frac{f^{(5)}(a)}{5!} =$	$c_5 \cdot (x-a)^5 =$
...	...	...	...
$f^{(n)}(x) =$	$f^{(n)}(a) =$	$c_n = \frac{f^{(n)}(a)}{n!} =$	$c_n \cdot (x-a)^n =$

Then $f(x) = \sum_{n=0}^{\infty} c_n (x-a)^n$

(1) Derive the first six terms of the Maclaurin series for $f(x) = \frac{1}{1-x}$:

$f(x) =$	$f(0) =$	$c_0 = \frac{f(0)}{0!} =$	$c_0 \cdot x^0 =$
$f'(x) =$	$f'(0) =$	$c_1 = \frac{f'(0)}{1!} =$	$c_1 \cdot x^1 =$
$f''(x) =$	$f''(0) =$	$c_2 = \frac{f''(0)}{2!} =$	$c_2 \cdot x^2 =$
$f'''(x) =$	$f'''(0) =$	$c_3 = \frac{f'''(0)}{3!} =$	$c_3 \cdot x^3 =$
$f^{(4)}(x) =$	$f^{(4)}(0) =$	$c_4 = \frac{f^{(4)}(0)}{4!} =$	$c_4 \cdot x^4 =$
$f^{(5)}(x) =$	$f^{(5)}(0) =$	$c_5 = \frac{f^{(5)}(0)}{5!} =$	$c_5 \cdot x^5 =$
...	...	...	...
$f^{(n)}(x) =$	$f^{(n)}(0) =$	$c_n = \frac{f^{(n)}(0)}{n!} =$	$c_n \cdot x^n =$

(2) Write out the first six terms of the Maclaurin series:

$$f(x) = \frac{1}{1-x} =$$

(3) What is the formula in summation form for the infinite Maclaurin series?

$$f(x) = \frac{1}{1-x} =$$

(4) What is its interval of convergence? Start by using the Ratio Test:

...., then check the endpoints of the interval separately only if necessary:

(1) Derive the first six terms of the Maclaurin series for $f(x) = e^x$:

$f(x) =$	$f(0) =$	$c_0 = \frac{f(0)}{0!} =$	$c_0 \cdot x^0 =$
$f'(x) =$	$f'(0) =$	$c_1 = \frac{f'(0)}{1!} =$	$c_1 \cdot x^1 =$
$f''(x) =$	$f''(0) =$	$c_2 = \frac{f''(0)}{2!} =$	$c_2 \cdot x^2 =$
$f'''(x) =$	$f'''(0) =$	$c_3 = \frac{f'''(0)}{3!} =$	$c_3 \cdot x^3 =$
$f^{(4)}(x) =$	$f^{(4)}(0) =$	$c_4 = \frac{f^{(4)}(0)}{4!} =$	$c_4 \cdot x^4 =$
$f^{(5)}(x) =$	$f^{(5)}(0) =$	$c_5 = \frac{f^{(5)}(0)}{5!} =$	$c_5 \cdot x^5 =$
...	...	...	...
$f^{(n)}(x) =$	$f^{(n)}(0) =$	$c_n = \frac{f^{(n)}(0)}{n!} =$	$c_n \cdot x^n =$

(2) Write out the first six terms of the Maclaurin series:

$f(x) = e^x =$

(3) What is the formula in summation form for the infinite Maclaurin series?

$f(x) = e^x$ =

(4) What is its interval of convergence? Start by using the Ratio Test:

...., then check the endpoints of the interval separately only if necessary:

(1) Derive the first six terms of the Maclaurin series for $f(x) = \sin(x)$:

$f(x) =$	$f(0) =$	$c_0 = \frac{f(0)}{0!} =$	$c_0 \cdot x^0 =$
$f'(x) =$	$f'(0) =$	$c_1 = \frac{f'(0)}{1!} =$	$c_1 \cdot x^1 =$
$f''(x) =$	$f''(0) =$	$c_2 = \frac{f''(0)}{2!} =$	$c_2 \cdot x^2 =$
$f'''(x) =$	$f'''(0) =$	$c_3 = \frac{f'''(0)}{3!} =$	$c_3 \cdot x^3 =$
$f^{(4)}(x) =$	$f^{(4)}(0) =$	$c_4 = \frac{f^{(4)}(0)}{4!} =$	$c_4 \cdot x^4 =$
$f^{(5)}(x) =$	$f^{(5)}(0) =$	$c_5 = \frac{f^{(5)}(0)}{5!} =$	$c_5 \cdot x^5 =$
...	...	...	...
$f^{(n)}(x) =$ _______ if n is even. $f^{(n)}(x) =$ _______ if n is odd.	$f^{(n)}(0) =$ ___ if n is even. $f^{(n)}(0) =$ ___ if n is odd.	$c_n = \frac{f^{(n)}(0)}{n!} =$ don't bother!	$c_n \cdot x^n =$ don't bother!

(2) Write out the first six non-zero terms of the Maclaurin series:

$f(x) = \sin(x) =$

(3) What is the formula in summation form for the infinite Maclaurin series?

$f(x) = \sin(x) \; =$

(4) What is its interval of convergence? Start by using the Ratio Test:

...., then check the endpoints of the interval separately only if necessary:

(1) Derive the first six terms of the Maclaurin series for $f(x) = \cos(x)$:

$f(x) =$	$f(0) =$	$c_0 = \dfrac{f(0)}{0!} =$	$c_0 \cdot x^0 =$
$f'(x) =$	$f'(0) =$	$c_1 = \dfrac{f'(0)}{1!} =$	$c_1 \cdot x^1 =$
$f''(x) =$	$f''(0) =$	$c_2 = \dfrac{f''(0)}{2!} =$	$c_2 \cdot x^2 =$
$f'''(x) =$	$f'''(0) =$	$c_3 = \dfrac{f'''(0)}{3!} =$	$c_3 \cdot x^3 =$
$f^{(4)}(x) =$	$f^{(4)}(0) =$	$c_4 = \dfrac{f^{(4)}(0)}{4!} =$	$c_4 \cdot x^4 =$
$f^{(5)}(x) =$	$f^{(5)}(0) =$	$c_5 = \dfrac{f^{(5)}(0)}{5!} =$	$c_5 \cdot x^5 =$
...	...	...	...
$f^{(n)}(x) =$ _______ if n is even. $f^{(n)}(x) =$ _______ if n is odd.	$f^{(n)}(0) =$ ___ if n is even. $f^{(n)}(0) =$ ___ if n is odd.	$c_n = \dfrac{f^{(n)}(0)}{n!} =$ don't bother!	$c_n \cdot x^n =$ don't bother!

(2) Write out the first six <u>non-zero</u> terms of the Maclaurin series:

$f(x) = \cos(x) =$

(3) What is the formula in summation form for the infinite Maclaurin series?

$f(x) = \cos(x)$ =

(4) What is its interval of convergence? Start by using the Ratio Test:

...., then check the endpoints of the interval separately only if necessary:

(1) Derive the first six terms of the Maclaurin series for $f(x) = \ln(x+1)$:

$f(x) =$	$f(0) =$	$c_0 = \frac{f(0)}{0!} =$	$c_0 \cdot x^0 =$
$f'(x) =$	$f'(0) =$	$c_1 = \frac{f'(0)}{1!} =$	$c_1 \cdot x^1 =$
$f''(x) =$	$f''(0) =$	$c_2 = \frac{f''(0)}{2!} =$	$c_2 \cdot x^2 =$
$f'''(x) =$	$f'''(0) =$	$c_3 = \frac{f'''(0)}{3!} =$	$c_3 \cdot x^3 =$
$f^{(4)}(x) =$	$f^{(4)}(0) =$	$c_4 = \frac{f^{(4)}(0)}{4!} =$	$c_4 \cdot x^4 =$
$f^{(5)}(x) =$	$f^{(5)}(0) =$	$c_5 = \frac{f^{(5)}(0)}{5!} =$	$c_5 \cdot x^5 =$
...	...	...	...
$f^{(n)}(x) =$ (if $n \geq 1$)	$f^{(n)}(0) =$	$c_n = \frac{f^{(n)}(0)}{n!} =$	$c_n \cdot x^n =$

(2) Write out the first six non-zero terms of the Maclaurin series:

$f(x) = \ln(x+1) =$

(3) What is the formula in summation form for the infinite Maclaurin series?

$f(x) = \ln(x+1)$ =

(4) What is its interval of convergence? Start by using the Ratio Test:

...., then check the endpoints of the interval separately only if necessary:

Write out a summary of your results, writing each Maclaurin series in both expanded and summation form.

original function	summation form	expanded form	interval of convergence
$\frac{1}{1-x}$	$\sum_{n=0}^{\infty}$		
e^x	$\sum_{n=0}^{\infty}$		
$\sin(x)$	$\sum_{k=0}^{\infty}$		
$\cos(x)$	$\sum_{k=0}^{\infty}$		
$\ln(x+1)$	$\sum_{n=1}^{\infty}$		