The Alpha and Beta of Risk Attribution

Jose Menchero
Outline

- Portfolio Analytics Overview
- Portfolio Optimization
- Attributing Risk to Alpha and Beta
  - Security Level
  - Brinson Model
  - Factor Approach
- Residual Weights versus Residual Returns
- Summary

Portfolio Analytics Overview
Three Basic Questions of Portfolio Management

Portfolio Managers are concerned with three basic questions:

1. How much does an investment decision contribute to return?
2. How much does an investment decision contribute to risk?
3. How much does an investment decision contribute to risk-adjusted return?
## Segmenting the Portfolio Analytics Space

<table>
<thead>
<tr>
<th>Return</th>
<th>Risk</th>
<th>Risk/Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alpha Modeling</td>
<td>Risk Forecasting</td>
<td>Portfolio Optimization</td>
</tr>
<tr>
<td></td>
<td>Risk Attribution</td>
<td></td>
</tr>
<tr>
<td>Performance Attribution</td>
<td>Ex Post Risk Attribution</td>
<td>Risk-adjusted Attribution</td>
</tr>
<tr>
<td></td>
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</tr>
</tbody>
</table>
Risk and Performance Attribution (Ex Post)

\[ Q_m = \sum_t \beta_t Q_{mt} \quad \rightarrow \quad R = \sum_m Q_m \]

Return Attribution

Multi-period linking coefficient (a)

\[ \sigma(R) = \sum_m \sigma(Q_m) \rho(Q_m, R) \]

Risk Attribution (b)

- Volatilities and correlations computed using standard time-series methods

Risk-Adjusted Attribution (Ex Post)

\[
\frac{R}{\sigma(R)} = \sum_{m} \left( \frac{\sigma(Q_m) \rho(Q_m, R)}{\sigma(R)} \right) \left( \frac{Q_m}{\sigma(Q_m) \rho(Q_m, R)} \right)
\]

“Risk Weight”

“Component IR”

- Analyzes each return source on a risk-adjusted basis
- Ideally, every source has the same component Information Ratio
- Answers how efficiently the risk budget was allocated in practice

Performance Attribution (*Ex Ante*)

\[
R = \sum_{m} \chi_m g_m
\]

\[
\chi_m = \text{Source Exposure}
\]

\[
g_m = \text{Source Return}
\]

Specific Examples:

\[
R_A = \sum_{n} (w_n^P - w_n^B)(r_n - R_B)
\]

Security Level

\[
R_A = \sum_{i} (w_i^P - w_i^B)(R_i^B - R_B) + \sum_{i} w_i^P (R_i^P - R_i^B)
\]

Brinson Model

\[
R_A = \sum_{k} X_k^A f_k + \sum_{n} w_n^A u_n
\]

Factor Approach
Risk Attribution (Ex Ante)

\[
\sigma(R) = \sum_{m} x_m \sigma(g_m) \rho(g_m, R)
\]

- Align risk and return attribution variables
- Risk contributions are additive and intuitive
- Identifies three drivers of portfolio risk:
  - Sizes of the exposures \( x_m \)
  - Stand-alone volatilities of the return sources \( \sigma(g_m) \)
  - Correlation of return sources with portfolio \( \rho(g_m, R) \)
- Volatility and correlation forecasts obtained from risk model

Information Ratio Attribution (Ex Ante)

\[ IR = \frac{R}{\sigma(R)}; \quad R = \sum_m x_m g_m; \quad \sigma(R) = \sum_m x_m \sigma(g_m) \rho(g_m, R) \]

\[ IR = \sum_m \left( \frac{x_m \sigma(g_m) \rho(g_m, R)}{\sigma(R)} \right) \left( \frac{x_m g_m}{x_m \sigma(g_m) \rho(g_m, R)} \right) \]

- Portfolio IR is the risk-weighted average of component IR
- Component IR is the stand-alone IR of return source, but magnified by \( \rho^{-1} \). This represents a diversification benefit.
Implied Returns

- For an unconstrained optimal portfolio, each component IR must equal the portfolio IR:

\[
IR_m = \frac{g_m}{\sigma(g_m) \rho(g_m, R)} = IR
\]

- The expected return of any asset can be “reverse engineered” by computing its volatility and correlation with the optimal portfolio

\[
E[g_m] = IR \cdot \sigma(g_m) \cdot \rho(g_m, R) = \beta_m \cdot E[R]
\]

- Result reduces to CAPM when the efficient portfolio is the Market

- Constraints reduce the portfolio IR by forcing part of the risk budget to be allocated to low IR positions
Residual Volatility and Transfer Coefficient

- Investment constraints force the *actual* portfolio $P$ to deviate from the *ideal* portfolio $Q$

\[ R_P = \beta_P R_Q + R_\varepsilon \]

- Residual portfolio contributes to risk, but *not* to expected return

- Compute betas and correlations relative to ideal portfolio $Q$

\[ \sigma^2_\varepsilon = \sigma_P \sqrt{1 - \rho^2} \]

Residual Volatility

\[ IR_P = \rho \cdot IR_Q \]

Transfer Coefficient
Portfolio Optimization
Unconstrained Optimal Portfolios

Minimize tracking error:  \[ \sum_{mn} w_m^A V_{mn} w_n^A \]

Subject to fixed return constraint:  \[ \sum_n w_n^A r_n = 1 \]

where:  \[ r_n = \alpha_n + \beta_n R_B \]

Solution:
\[
w_A = \frac{V^{-1}r}{r'V^{-1}r}
\]

Optimal Portfolio
\[
w_n^A = \frac{\sum_m r_m V_{mn}^{-1}}{\sum_{mn} r_m V_{mn}^{-1} r_n}
\]

- Optimal portfolio has maximum Information Ratio
- Fixed return constraint determines leverage, does not impact IR
- In general, active weights do not sum to zero
- In general, active beta is non-zero
Optimal Portfolios with Constraints

Fixed return constraint: \[
\begin{pmatrix}
  r_1 & r_2 & r_3 & \cdots & r_N
\end{pmatrix}
\begin{pmatrix}
  w_1^A \\
  w_2^A \\
  \vdots \\
  w_N^A
\end{pmatrix} = \begin{pmatrix}
  1 \\
  \end{pmatrix}
\]

Full investment constraint: \[
\begin{pmatrix}
  1 & 1 & 1 & \cdots & 1
\end{pmatrix}
\begin{pmatrix}
  \beta_1 & \beta_2 & \beta_3 & \cdots & \beta_N
\end{pmatrix}
\begin{pmatrix}
  w_1^A \\
  w_2^A \\
  \vdots \\
  w_N^A
\end{pmatrix} = \begin{pmatrix}
  0 \\
  \end{pmatrix}
\]

Zero beta constraint: \[
\begin{pmatrix}
  \beta_1 & \beta_2 & \beta_3 & \cdots & \beta_N
\end{pmatrix}
\begin{pmatrix}
  w_1^A \\
  w_2^A \\
  \vdots \\
  w_N^A
\end{pmatrix} = \begin{pmatrix}
  0 \\
  \end{pmatrix}
\]

Minimize tracking error: \[ w_A' V w_A \]

Subject to constraint equation: \[ Q w_A = c \]

Analytic Solution: \[ w_A = V^{-1} Q' \left( QV^{-1} Q' \right)^{-1} c \]

More complex constraints require numerical solution
Attributing Risk to Alpha and Beta
Segmenting Source Returns into Alpha and Beta

\( g_m = g_m^\alpha + g_m^\beta \)

Decompose Return Source into Alpha and Beta

\( g_m^\beta = \beta_m \cdot R_B \)

Beta component is perfectly correlated with the benchmark

\( g_m^\alpha = g_m - g_m^\beta \)

Alpha component is uncorrelated with the benchmark

\[
\sigma(R_A) = \sum_m x_m \sigma(g_m^\alpha) \rho(g_m^\alpha, R_A) + \sum_m x_m \sigma(g_m^\beta) \rho(g_m^\beta, R_A)
\]

Alpha Risk
Beta Risk

- Now compute betas relative to benchmark
- Note: active strategy presupposes benchmark is not optimal
Example

- Portfolio: 95% MSCI World Value Index, with 5% cash (USD)
- Benchmark: MSCI World Growth Index
- Risk Model: Barra Global Equity Model GEM2L
- Analysis Date: September 30, 2009
- Portfolio Beta: 1.08
- Tracking Error: 4.81%

\[ R_A = \alpha_P + \beta_A R_B \]

<table>
<thead>
<tr>
<th>Source</th>
<th>Volatility</th>
<th>Correlation</th>
<th>TE Contrib</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alpha</td>
<td>4.32%</td>
<td>0.90</td>
<td>3.89%</td>
</tr>
<tr>
<td>Beta</td>
<td>2.10%</td>
<td>0.44</td>
<td>0.92%</td>
</tr>
<tr>
<td>Total</td>
<td>4.81%</td>
<td>1.00</td>
<td>4.81%</td>
</tr>
</tbody>
</table>

- Assuming optimality and \( IR=1 \): \( E[R_B] = \left( \frac{0.92}{0.08} \right) = 11.8\% \)
Security Level

- Consider bottom-up investment process:

\[ R_A = \sum_n w^A_n (r_n - R_B) \]

- Relative returns are more appropriate than absolute returns
- Intuitively, we want to overweight the outperformers, but sometimes it is optimal to *underweight* outperformers (hedging purposes)
- The benchmark represents the risk-free asset (zero relative volatility)
- Cash has relative volatility \( \sigma_B \)
- Relative return of Cash has correlation of -1.0 to the benchmark
### Example: Security Level

<table>
<thead>
<tr>
<th>Asset Name</th>
<th>Active Weight</th>
<th>Relative Volatility</th>
<th>Relative Corr</th>
<th>TE Contrib</th>
<th>Alpha Component</th>
<th>Beta Component</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Relative Volatility</td>
<td>Relative Corr</td>
<td>TE Contrib</td>
</tr>
<tr>
<td>BANK OF AMERICA</td>
<td>1.31%</td>
<td>61.77%</td>
<td>0.44</td>
<td>0.35%</td>
<td>57.12%</td>
<td>0.29</td>
<td>0.22%</td>
</tr>
<tr>
<td>CITIGROUP</td>
<td>0.69%</td>
<td>82.99%</td>
<td>0.41</td>
<td>0.24%</td>
<td>75.48%</td>
<td>0.25</td>
<td>0.13%</td>
</tr>
<tr>
<td>GENERAL ELECTRIC</td>
<td>1.56%</td>
<td>35.56%</td>
<td>0.38</td>
<td>0.21%</td>
<td>33.29%</td>
<td>0.25</td>
<td>0.13%</td>
</tr>
<tr>
<td>JPMORGAN CHASE</td>
<td>1.47%</td>
<td>40.66%</td>
<td>0.37</td>
<td>0.22%</td>
<td>39.32%</td>
<td>0.26</td>
<td>0.15%</td>
</tr>
<tr>
<td>NESTLE</td>
<td>-1.52%</td>
<td>24.41%</td>
<td>-0.22</td>
<td>0.08%</td>
<td>22.60%</td>
<td>-0.06</td>
<td>0.02%</td>
</tr>
<tr>
<td>...</td>
<td></td>
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<td></td>
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<td></td>
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</tr>
<tr>
<td>UBS</td>
<td>-0.64%</td>
<td>41.47%</td>
<td>0.26</td>
<td>-0.07%</td>
<td>38.49%</td>
<td>0.11</td>
<td>-0.03%</td>
</tr>
<tr>
<td>RIO TINTO</td>
<td>-0.57%</td>
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<td>0.07</td>
<td>-0.02%</td>
<td>43.69%</td>
<td>-0.12</td>
<td>0.03%</td>
</tr>
<tr>
<td>EXXON MOBIL</td>
<td>3.00%</td>
<td>23.64%</td>
<td>0.06</td>
<td>0.04%</td>
<td>23.16%</td>
<td>0.15</td>
<td>0.10%</td>
</tr>
<tr>
<td>BHP BILLITON</td>
<td>-1.10%</td>
<td>39.76%</td>
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<td>-0.01%</td>
<td>36.70%</td>
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</tr>
<tr>
<td>US Dollar</td>
<td>5.00%</td>
<td>26.78%</td>
<td>-0.44</td>
<td>-0.59%</td>
<td>0.00%</td>
<td>0.00</td>
<td>0.00%</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>4.81%</td>
<td></td>
<td>3.89%</td>
</tr>
</tbody>
</table>

- Assuming optimality and $IR=1$, expected return of benchmark is $(26.78)(0.44)$, or 11.8 percent.
- GE is expected to outperform benchmark by $(35.56)(0.38)$, or 13.5 percent.
- Portfolio is underweight UBS, although it has a positive expected return.
- US Dollar is the greatest diversifier and contributes zero to Alpha risk.
Example: Sector-Based Approach

Brinson Model:

\[ R = \sum_i (w_i^P - w_i^B)(R_i^B - R_B) + \sum_i w_i^P (R_i^P - R_i^B) \]

<table>
<thead>
<tr>
<th>Sector Name</th>
<th>Portfolio Weight</th>
<th>Bench Weight</th>
<th>Active Weight</th>
<th>Relative Volatility</th>
<th>Relative Corr</th>
<th>Allocation TE Contrib</th>
<th>Active Volatility</th>
<th>Active Corr</th>
<th>Selection TE Contrib</th>
<th>Total TE Contrib</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy</td>
<td>13.09%</td>
<td>7.77%</td>
<td>5.32%</td>
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<td>Materials</td>
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<td>-0.01%</td>
<td>12.09%</td>
<td>0.41</td>
<td>0.25%</td>
<td>0.23%</td>
</tr>
<tr>
<td>Industrials</td>
<td>10.55%</td>
<td>9.85%</td>
<td>0.70%</td>
<td>5.80%</td>
<td>0.32</td>
<td>0.01%</td>
<td>7.84%</td>
<td>0.45</td>
<td>0.38%</td>
<td>0.39%</td>
</tr>
<tr>
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<td>11.27%</td>
<td>-4.31%</td>
<td>7.50%</td>
<td>0.30</td>
<td>-0.10%</td>
<td>6.44%</td>
<td>0.25</td>
<td>0.11%</td>
<td>0.01%</td>
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<td>4.39%</td>
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<td>8.15%</td>
<td>0.26</td>
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<td>1.73%</td>
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<td>0.57</td>
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</tr>
<tr>
<td>IT</td>
<td>3.25%</td>
<td>19.95%</td>
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<td>-0.16</td>
<td>0.25%</td>
<td>10.34%</td>
<td>0.08</td>
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<td>0.28%</td>
</tr>
<tr>
<td>Telecom</td>
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<td>3.36%</td>
<td>14.16%</td>
<td>-0.02</td>
<td>-0.01%</td>
<td>14.39%</td>
<td>0.10</td>
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<tr>
<td>Utilities</td>
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<td>2.26%</td>
<td>4.47%</td>
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<td>-0.17</td>
<td>-0.10%</td>
<td>9.28%</td>
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<td>0.16%</td>
<td>0.05%</td>
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<tr>
<td>Cash</td>
<td>5.00%</td>
<td>0.00%</td>
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<td>26.78%</td>
<td>-0.44</td>
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<tr>
<td>Total</td>
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<td>100.00%</td>
<td>0.00%</td>
<td>3.94%</td>
<td>0.47</td>
<td>1.84%</td>
<td>4.57%</td>
<td>0.65</td>
<td>2.96%</td>
<td>4.81%</td>
</tr>
</tbody>
</table>

- Financial sector contributes 350 bps to TE; 173 bps due to Allocation
- Each attribution effect can be subdivided into Alpha and Beta components
### Alpha Decomposition for Brinson Model

<table>
<thead>
<tr>
<th>Sector Name</th>
<th>Portfolio Weight</th>
<th>Bench Weight</th>
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</tr>
<tr>
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<td>0.00%</td>
<td>0.00%</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>100.00%</strong></td>
<td><strong>100.00%</strong></td>
<td><strong>0.00%</strong></td>
<td><strong>3.81%</strong></td>
<td><strong>0.37</strong></td>
<td><strong>1.41%</strong></td>
<td><strong>4.43%</strong></td>
<td><strong>0.56</strong></td>
<td><strong>2.48%</strong></td>
<td><strong>3.89%</strong></td>
</tr>
</tbody>
</table>

- **Alpha component contributes 108 bps (of 173 bps total) to Financials Allocation**
## Beta Decomposition for Brinson Model

<table>
<thead>
<tr>
<th>Sector Name</th>
<th>Portfolio Weight</th>
<th>Bench Weight</th>
<th>Active Weight</th>
<th>Beta (Market Timing) Component</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Relative Volatility</td>
<td>Relative Corr</td>
<td>Allocation TE Contrib</td>
<td>Active Volatility</td>
</tr>
<tr>
<td>Energy</td>
<td>13.09%</td>
<td>7.77%</td>
<td>5.32%</td>
<td>8.44%</td>
</tr>
<tr>
<td>Materials</td>
<td>4.98%</td>
<td>8.80%</td>
<td>-3.82%</td>
<td>7.98%</td>
</tr>
<tr>
<td>Industrials</td>
<td>10.55%</td>
<td>9.85%</td>
<td>0.70%</td>
<td>1.46%</td>
</tr>
<tr>
<td>Cons Disc</td>
<td>6.97%</td>
<td>11.27%</td>
<td>-4.31%</td>
<td>1.78%</td>
</tr>
<tr>
<td>Cons Stpls</td>
<td>4.39%</td>
<td>15.65%</td>
<td>-11.25%</td>
<td>6.94%</td>
</tr>
<tr>
<td>Health Care</td>
<td>6.53%</td>
<td>13.49%</td>
<td>-6.96%</td>
<td>6.42%</td>
</tr>
<tr>
<td>Financials</td>
<td>32.57%</td>
<td>8.39%</td>
<td>24.19%</td>
<td>6.08%</td>
</tr>
<tr>
<td>IT</td>
<td>3.25%</td>
<td>19.95%</td>
<td>-16.70%</td>
<td>0.03%</td>
</tr>
<tr>
<td>Telecom</td>
<td>5.95%</td>
<td>2.58%</td>
<td>3.36%</td>
<td>3.29%</td>
</tr>
<tr>
<td>Utilities</td>
<td>6.72%</td>
<td>2.26%</td>
<td>4.47%</td>
<td>7.50%</td>
</tr>
<tr>
<td>Cash</td>
<td>5.00%</td>
<td>0.00%</td>
<td>5.00%</td>
<td>26.78%</td>
</tr>
<tr>
<td>Total</td>
<td>100.00%</td>
<td>100.00%</td>
<td>0.00%</td>
<td>1.00%</td>
</tr>
</tbody>
</table>

- Beta component contributes 64 bps (of 173 bps total) to Financials Allocation
- Correlation between benchmark and active portfolio is 0.44
Factor-Based Approach

- Attribute risk to a set of custom factors:
  - World factor
  - 10 GICS Economic Sectors
  - Use Predicted Beta as a style factor, together with Value, Size, and Momentum
  - Style factors are cap-weighted mean zero
  - Use benchmark as estimation universe
  - Use benchmark cap-weights as regression weights

- All factor portfolios (except Beta and World) have zero beta
- World factor portfolio becomes the benchmark (beta=1)
- Beta factor portfolio is dollar neutral (also with beta=1)

\[ R = \sum_{k} X^P_k f_k + \sum_{n} w_n u_n \]
Example: Factor-Based Approach

- Beta risk is fully explained by Beta and World factors
- World factor accounts for hedging due to cash position (-59 bps)
- Beta factor portfolio contains some residual risk
Residual Weights versus Residual Returns
Alpha Analysis: Residual Weights or Residual Returns?

- Our approach is based on active weights and residual returns
  - Active weights are intuitive
  - Residual returns are uncorrelated with the benchmark

- Alternative approach: use residual weights with total returns

- This adds up to the total portfolio alpha:
  \[ \sum_n (w_n^P - \beta_P w_n^B)(\alpha_n + \beta_n R_B) = \alpha_P \]

- Shortcomings:
  - Residual weights are less intuitive
  - Total returns are correlated with the benchmark
  - Inconsistent with notion of Alpha as an uncorrelated return source
Example: Residual Exposures

<table>
<thead>
<tr>
<th>Source</th>
<th>Residual Exposure</th>
<th>Factor Volatility</th>
<th>Corr</th>
<th>TE Contrib</th>
</tr>
</thead>
<tbody>
<tr>
<td>World</td>
<td>-0.13</td>
<td>26.78%</td>
<td>0.44</td>
<td>-1.51%</td>
</tr>
<tr>
<td>Energy</td>
<td>0.05</td>
<td>16.20%</td>
<td>-0.08</td>
<td>-0.06%</td>
</tr>
<tr>
<td>Materials</td>
<td>-0.05</td>
<td>14.59%</td>
<td>-0.20</td>
<td>0.13%</td>
</tr>
<tr>
<td>Industrials</td>
<td>0.00</td>
<td>5.34%</td>
<td>0.20</td>
<td>0.00%</td>
</tr>
<tr>
<td>ConsDscr</td>
<td>-0.05</td>
<td>7.30%</td>
<td>0.22</td>
<td>-0.08%</td>
</tr>
<tr>
<td>ConsStpls</td>
<td>-0.12</td>
<td>7.17%</td>
<td>0.03</td>
<td>-0.03%</td>
</tr>
<tr>
<td>HealthCare</td>
<td>-0.08</td>
<td>8.89%</td>
<td>-0.12</td>
<td>0.08%</td>
</tr>
<tr>
<td>Financials</td>
<td>0.24</td>
<td>11.92%</td>
<td>0.40</td>
<td>1.12%</td>
</tr>
<tr>
<td>IT</td>
<td>-0.18</td>
<td>9.12%</td>
<td>-0.13</td>
<td>0.22%</td>
</tr>
<tr>
<td>Telecom</td>
<td>0.03</td>
<td>14.07%</td>
<td>0.06</td>
<td>0.03%</td>
</tr>
<tr>
<td>Utilities</td>
<td>0.04</td>
<td>10.62%</td>
<td>0.00</td>
<td>0.00%</td>
</tr>
<tr>
<td>Momentum</td>
<td>0.05</td>
<td>6.28%</td>
<td>-0.12</td>
<td>-0.04%</td>
</tr>
<tr>
<td>Beta</td>
<td>0.13</td>
<td>31.57%</td>
<td>0.36</td>
<td>1.48%</td>
</tr>
<tr>
<td>Value</td>
<td>0.48</td>
<td>4.74%</td>
<td>0.14</td>
<td>0.32%</td>
</tr>
<tr>
<td>Size</td>
<td>0.07</td>
<td>3.05%</td>
<td>-0.13</td>
<td>-0.03%</td>
</tr>
<tr>
<td>Specific</td>
<td>1.00</td>
<td>4.02%</td>
<td>0.56</td>
<td>2.27%</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
<td>3.89%</td>
</tr>
</tbody>
</table>

- Factor exposures are quoted on a residual basis
- World and Beta factors make non-intuitive contribution to Alpha risk

\[
X_k^R = \sum_n w_n^R X_{nk}
\]

Residual Exposures
Summary

- Portfolio returns can always be segmented into Alpha and Beta:
  - Alpha component is uncorrelated with the benchmark
  - Beta component is perfectly correlated with the benchmark
- Portfolio risk can always be segmented into Alpha and Beta sources
- Three examples:
  - Security Level
  - Brinson Model
  - Factor Approach
- Alternative formulation based on residual weights leads to non-intuitive results
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<table>
<thead>
<tr>
<th>Americas</th>
<th>Europe, Middle East &amp; Africa</th>
<th>Asia Pacific</th>
</tr>
</thead>
<tbody>
<tr>
<td>Americas</td>
<td>Cape Town +27.21.673.0100</td>
<td>China North 10800.852.1032 (toll free)</td>
</tr>
<tr>
<td>Atlanta +1.404.551.3212</td>
<td>Frankfurt +49.69.133.859.00</td>
<td>China South 10800.152.1032 (toll free)</td>
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<tr>
<td>Boston +1.617.532.0920</td>
<td>Geneva +41.22.817.9777</td>
<td>Hong Kong +852.2844.9333</td>
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<tr>
<td>Chicago +1.312.706.4999</td>
<td>London +44.20.7618.2222</td>
<td>Seoul +798.8521.3392 (toll free)</td>
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<tr>
<td>Monterrey +52.81.1253.4020</td>
<td>Milan +39.02.5849.0415</td>
<td>Singapore 800.852.3749 (toll free)</td>
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</table>

**clientservice@msci.com**

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