

Scientific and Algebraic Thinking: Visualizing with Interactive Excel and Nested Styrofoam Cups

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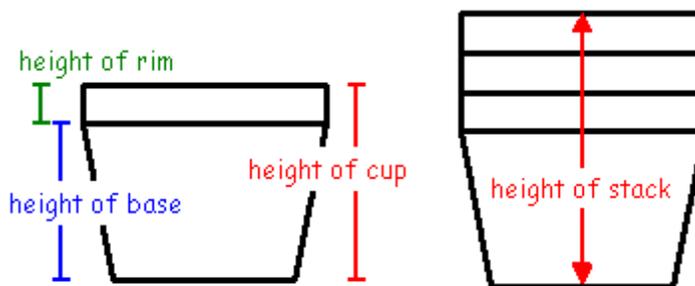
Abstract

Using the stacking of nested Styrofoam cups, we construct a mathematical model and explore it through simulations, all in Excel (prior experience unnecessary). This paper connects mathematics to measurement and measurement error via a “click-and-think” engaging pedagogy using predict, test, and analyze by means of an interactive Excel spreadsheet or Excelet. The “how to” aspects and resources are provided at <http://academic.pgcc.edu/~ssinex/excelets>. We examine questions for student assessment of the model and error as well.

Introduction

Can you start mathematical modeling on the first day of class? Yes! This is a cooperative group activity using a “just add data” interactive Excel spreadsheet to build a linear mathematical model and examine it mathematically and scientifically. This is achieved by examining the equation of the model and relating to the animated physical situation used by simulation as students predict, test, and analyze and then explain their observations. The activity is aligned with the CRAFTY College Algebra Guidelines [1]. A variety of errors, both random and systematic, and their effect on the mathematical model will be explored via simulation in Excel, all done computationally (no programming or macros required). The “rule of four” is elegantly demonstrated with an engaging pedagogy that is enhanced by off-the-shelf, work-place common technology (a big advantage for K-12 teachers). Post-activity assessment examples are also provided. This mathematical model is well-behaved due to the consistent manufacture of most cups and with induced error represents real-world scientific analysis. Both measurement error and manufacturing error are considered. Developing a mind-set to examine error in measurement is important for students, as is writing equations in terms of the variables studied (not x and y).

Linear regression is used to generate the line of best-fit; however, knowledge of regression is not needed nor is prior Excel knowledge required to use the spreadsheet. Students will measure the stack height of nested Styrofoam cups as illustrated to the right along with some terminology used in describing the cups.



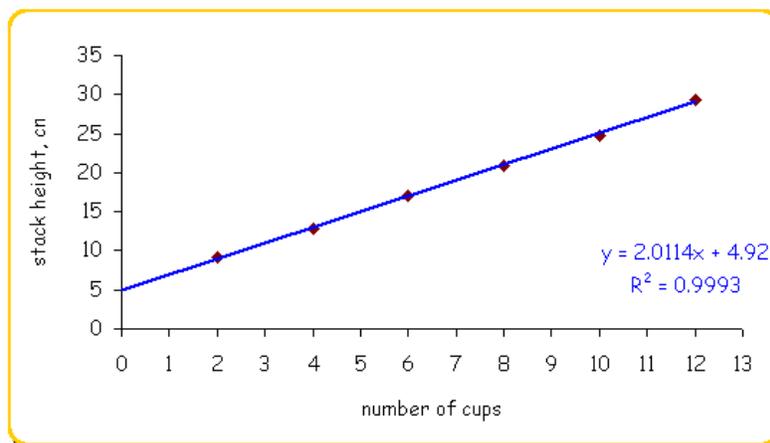
Constructing the Mathematical Model

Students (and readers) will need to get the interactive Excel spreadsheet or Excelet at http://academic.pgcc.edu/~ssinex/excelets/stacking_cups.xls. Using a centimeter ruler, students will measure and record to the nearest 0.1 cm the height of a variety of stacks of nested Styrofoam cups. The data can be entered as collected in this “just add data” Excelet and the graph and regression line will appear and adjust as data is entered. This can be done without the knowledge of linear regression. Students just need to know that the line is the “line of best fit” and how to judge this from scatter in the data, where use of R^2 helps. With careful measurement, this model is usually a very good fit of the data due to the consistent manufacturing of the Styrofoam cups. If instructors have groups use different cup volumes, a number of models can be produced for comparison.

Measuring the Stack Height of Nested Styrofoam Cups - Building a Mathematical Model

Using a centimeter ruler, carefully measure the stacks to the nearest 0.1 cm. Record your data in the yellow cells.

number of cups	stack height
2	9.1
4	12.8
6	17.1
8	20.9
10	24.8
12	29.3



From the best-fit line record the following in the yellow cells:

What is the value of the slope?

2.01

What is the value of the intercept?

4.92

How well does the line fit the data?

R^2 value

As data are added, the points will plot and a best-fit line and the equation of that line will appear on the graph.

Rewrite the equation in terms of the actual variables measured. answer

Let's explore what the linear equation means.

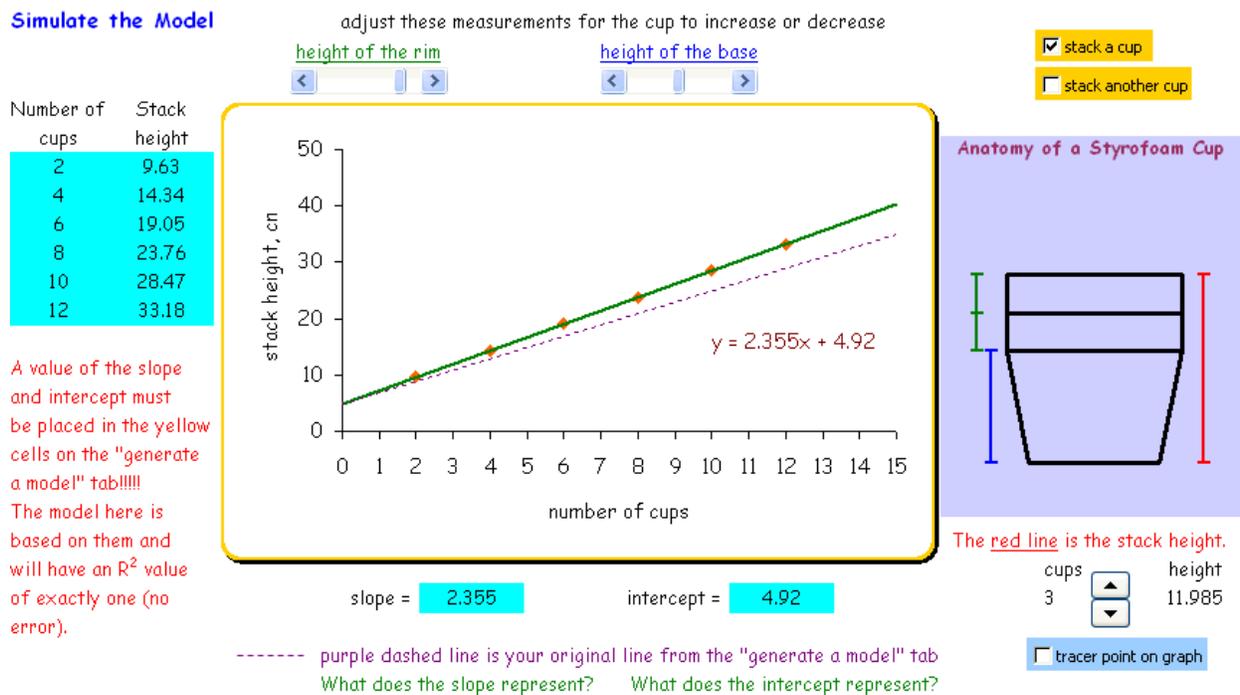
Let's predict using the equation

Once we have the mathematical model, students can rewrite the equation found in terms of the variables measured ($H = 2.01n + 4.92$ where n = number of cups and H = stack height), we check their knowledge of slope and intercept from the model (plus this feeds into other tabs on the spreadsheet), and we make predictions with the model. We can interpolate and extrapolate and even verify with measurement. These are all good science process skills for students to practice. Now, can we get students to understand this model? One observation that usually surfaces is why is there any height when I have zero cups!

Adding Simulation to the Model

Now we are ready to explore the variables in the mathematical model developed from the data. This tab of the spreadsheets contains a number of interactive features – the scroll bars to adjust rim and base heights, check boxes to stack cups or add a tracer point, and a spinner to predict height based on number of cups. The “Anatomy of a Styrofoam Cup” graphic to the right on the screen is simply a graph that shifts points when you check the “stack a cup” check box.

Simulate the Model



The scatter plot above is produced from the slope and intercept that was found from the experimental data. This produces an errorless set of data. At this point students could predict what the effect of changing the rim and base heights will do. Now we can adjust the height of the rim and height of the base to have students discover how they influence the graph. Students can quickly discover what controls the slope (height of the rim) and the y-intercept (height of the base). Students need to be able to clearly and correctly state how the variables influenced the regression line. This tab of the spreadsheet relates the mathematical model to the actual physical model. The fact that this model has a y-intercept is now easily grasped. The nesting of the Styrofoam cups produces this type of behavior which contrasts to the direct relationship found for the stacking of sandwich cookies [2] or any consistently sized object. Before considering error in the model, let's examine the inverse function and its use.

Generating the Inverse Function

From our experiment, the stack height of nested cups depends on the number of cups. Stack height is our dependent variable and number of cups is the independent variable. Mathematically, stack height is a function of the number of cups. The inverse function switches our variables so that the number of cups is a function of the stack height. This could be important when considering packaging stacked cups in a shipping container. On the inverse function tab, we use the experimentally collected data and reverse the order of the variables to plot the inverse function. The linear regression equation is generated directly on the graph, as well as a value of R^2 . The $y = x$ line can also be added to show that the inverse function is a reflection of the original function across the $y = x$ line. Students should see there is no difference in the goodness-of-fit as it's the same data as the original plot.

Generating the Inverse Function

copy data from "generating a model" tab

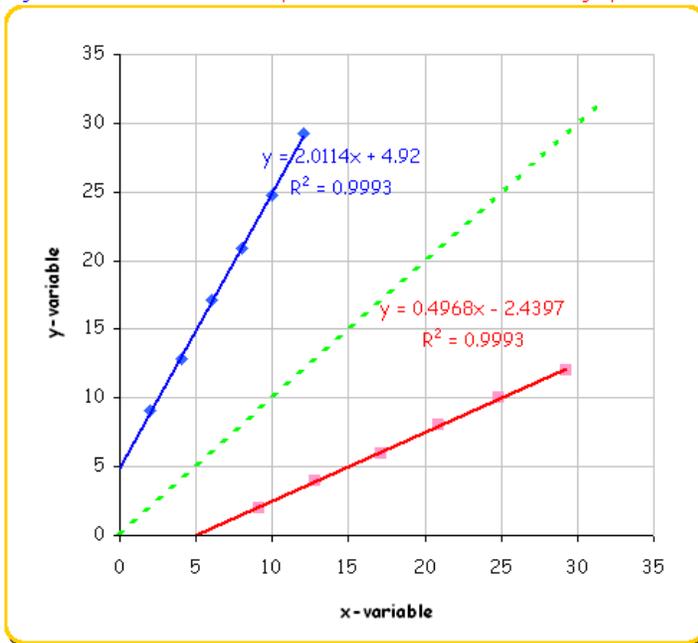
x	y
number of cups	stack height
2	9.1
4	12.8
6	17.1
8	20.9
10	24.8
12	29.3

The height of the stack depends on the number of cups.

Suppose we had shipping boxes of 61 cm and 100 cm, how many cups can we stack? copy each column over in reverse order here

Add the plot of the inverse function to the graph.

x	y
stack height	number of cups
9.1	2
12.8	4
17.1	6
20.9	8
24.8	10
29.3	12



add the y = x line

Write the equation of the inverse function in terms of the variables.

How does the goodness of fit behave for the inverse function?

Temperature conversion equations ($^{\circ}\text{F}$ to $^{\circ}\text{C}$ and $^{\circ}\text{C}$ to $^{\circ}\text{F}$) are another good example of inverse functions and a good way to introduce this concept. See [“The Temperature Scales on Thermometers: How are they related?”](#) Excelet and related activity to get students handling and graphing data. Now, let’s consider introducing error into our simulation and seeing what happens.

Simulating Error with the Model

In the real world of scientific measurement, experimental error is a very important item to consider. If you are in need of an introduction to the types of error, see Sinex [3] for a discussion and Sinex, Gage, and Beck [2] for an experimental approach using cookies. Both papers include an Excelet. Now for our Styrofoam cups model, we need to consider both random and systematic errors for measurements and manufacturing. Using the scroll bars with the various errors, students will discover how the errors influence the data and the resulting linear regression line and equation. The comment boxes on this tab give some further explanation about the errors.

Here is a summary of the effects of random and systematic errors.

Effects of the Errors

Errors	Slope of line	y-intercept	Scatter of data
Random measurement error	(small changes)	(small changes)	r^2 decreases –adds scatter
Systematic measurement error	No effect	Positive – intercept increases Negative - intercept decreases	No effect
Rim uniformity	(small changes)	(small changes)	r^2 decreases –adds scatter
Base uniformity single stack	No effect	Taller - intercept increases Shorter - intercept decreases	No effect
Base uniformity multiple stacks	(small changes)	(small changes)	r^2 decreases –adds scatter

Investigating the errors will prove to be a challenge to many students, especially if this is the first time they have really been confronted with it. Just handling the mathematical language is also a big consideration for many students. However, this is the real world of scientific analysis of experimental data and mathematical modeling. The screenshot below shows the systematic error or bias for the case of the ruler having the zero mark above the end of the ruler (very typical on 30 cm rulers) where all the readings are a constant factor low (~2 mm). Systematic errors induce a bias in either the positive or negative direction. Check your rulers and notice if your students check as well!

Simulate Error with the Model

Add each error, one at a time, to see how it influences the data and regression line. Set back to no error before going to the next error.

Number of cups	Stack height
2	9
4	13
6	17
8	21
10	25
12	29

$y = 2x + 5$
 $R^2 = 1$

ADD SOME ERROR:

Random Measurement Error
 increase ---->

Systematic Measurement Error
 negative 0 positive

Rim Uniformity
 high low

Base Uniformity
 single stack: (just bottom cup)
 shorter 0 taller
 multiple stacks: (bottom cups)
 high low

view error in table form

A value of the slope and intercept must be placed in the yellow cells on the "generate a model" tab!!!!
 The model here is based on them and will start with an R^2 value of exactly one.
So it is errorless !!!!!

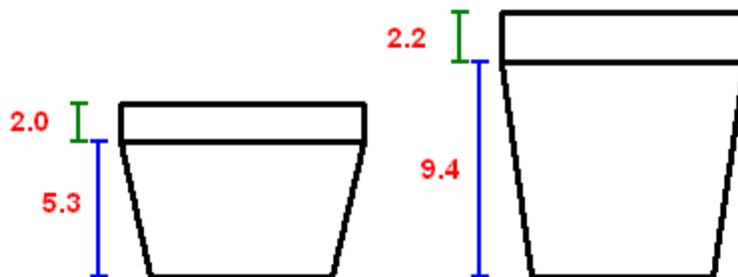
----- purple dashed line is your original line from the "generate a model" tab and we are assuming this data contains NO error.

slope =

intercept =

Assessment

A variety of student assessment questions are given on the assessment tab to check to see if students understand the mathematical model produced using the nested Styrofoam cups. The illustration shows two different Styrofoam cups, generate the mathematical models for each cup and plot a graph showing the models produced by both cups. Describe how the two graphs differ based on the cup parameters.



Based on an increase in cup volume, what can you conclude about how the rim height and base height behave? This adds another possible variable, volume, to the model.

Questions on the analysis of errors are another good category of question which includes combining errors [2] and understanding how an outlier datum point influences regression lines [4].

Constructing Interactive Excel Spreadsheets or Excelets

The wherewithal for producing Excelets can be found at the [Developer's Guide to Excelets](#) [5], which includes a tutorial, illustrated instructions, and many more examples. Take the [interactive features tour](#) to see what you can do in Excel. The forms toolbar provides a variety of features (spinners, scroll bars, checkboxes, etc.) that are easy to use, and when combined with logical functions, lookup tables, conditional formatting, and a number of simple tricks provides a wealth of interactivity and dynamic display. The use of the forms toolbar allows Excelets to function on both PC and Mac platforms. All of this is done using computations (a.k.a. - formulas and available functions) in the cells. The use of comment boxes adds explanation, hints, and answers for students as well. Always look under the graphs, as many of the tricks, such as turning lines on graphs on and off or tracer points are explained there. If you are just getting into Excel, see [Using Excel for Handling, Graphing, and Analyzing Scientific Data](#) [6] to gain experience and as a resource for your students.

The support materials for mathematical modeling are also given at the [Developer's Guide to Excelets](#) [5]. This includes modeling and simulations with cookies, introduction to linear regression, goodness of fit for linear models, interpolation and extrapolation, and non-linear models as well.

Some Final Thoughts

In this author's opinion, this is the "Right Stuff" and approach for college algebra as recommended by AMATYC [7]. It incorporates guided-inquiry or discovery learning with an interactive spreadsheet providing a very engaging pedagogy. It includes scientific and algebraic thinking and, with the inclusion of error analysis, is very real-world. This approach using off-

the-shelf software supports the “Rule of Four.” Through the mode of questioning used with the interactive Excel spreadsheet or Excelet students are constantly linking between the numerical, graphical, symbolic and verbal aspects. Excelets are also an excellent development tool for K-12 mathematics teacher education.

References

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4. Sinex, S.A., *Exploring the Goodness of Fit in Linear Models*, **Journal of Online Mathematics and its Application** 5 (2005).
5. Sinex, S.A., *Developer’s Guide to Excelets*, <http://academic.pgcc.edu/~ssinex/excelets> (2008).
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7. Kimball, R. *Revitalizing College Algebra*, <http://www.therightstuff.amatyc.org/RightStuff.ppt> (2008).