During the course of this three year project, the Principal Investigator (PI) Paul Seeburger has:

- Created the Multivariable Calculus Visualization applet using Java and published it on a website: web.monroecc.edu/calcNSF
- Developed a series of four guided exploration/assessments to be used by students.
- Given a 4-hour minicourse at MathFest 2009 & MathFest 2010 on the project.

For more information, email: pseeburger@monroecc.edu

Current Features of the CalcPlot3D applet, a Multivariable Calculus Exploration Environment:

- Up to four surface plots of functions of two variables can be graphed at the same time in Cartesian, cylindrical, or spherical coordinates. At this point, each function can be of the form: $$z = f(x, y), \ y = f(x, z), \ x = f(y, z), \ r = f(\theta, z), \ or \ \rho = f(\theta, \phi).$$ The real or imaginary part of complex functions of the form, $$z = f(w),$$ can also be graphed as surfaces over the complex plane.

- Multiple parametric space curves can be graphed simultaneously along with the surfaces. Motion along these space curves can be animated for a given time interval, showing the position vector and optionally, the velocity and/or acceleration vectors, the TNB-frame, the osculating circle, or the curvature value.

- Contour plots can be graphed in 2D and also in 3D on the surface. (A level plane can be moved through the surface to demonstrate that contours are cross-sections of the surface.)

- Vector fields can be graphed in both 2D and 3D. These can be laid out using a rectangular or circular grid in 2D and a rectangular, cylindrical, or spherical grid for 3D.

- Regions of integration can be created and graphed. First the 2D region is created using a dialog which shows the graph of the region in the xy-plane. The bottom and top (left and right) functions can be entered in either $$y = f(x)$$ form or $$x = f(y)$$ form, along with bounds on $$x$$ or $$y,$$ respectively. Once this region is created, the user can choose one of the four surfaces to use for a "top" for the solid region, and this solid region can be graphed on the 3D plot. An inner or outer partition of prisms can be displayed for any region.

- Parametric Surfaces can be graphed using two parameters. Numerous examples are provided.

- Implicit Surfaces (Implicit Equations) of the form, $$f(x, y, z) = c,$$ can be graphed. Examples are provided.

- Unlimited points and vectors can be added to the plot.

- Tangent planes and tangent lines (in the x- and y-directions) can be displayed and dynamically moved around the surface using the trace plane in the lower left corner of the applet.

- A gradient vector can be displayed in this trace plane and moved around a contour plot or surface.

- Taylor polynomials in two variables, $$P_n(x, y),$$ can be calculated and stepped through for any function, $$z = f(x, y).$$ Depending on the complexity of the partial derivatives, these can be viewed from degree 0 to as high a degree (less than 16) that the memory can handle for the given function.
VIEWING FEATURES:

- All 3D graphs can be rotated in real time in response to the movement of the mouse on the graph by the user. Graphs can be made to rotate in any direction to help students gain perspective.

- You can use 3D glasses for a more convincing 3D view of the 3D plots in CalcPlot3D. You can use Red-Cyan/Red-Blue 3D glasses for almost all of the options (except the Amber-Blue option) and there is an option for Red-Green 3D glasses. There are two options for viewing the images in 3D without 3D glasses: Stereo Pair and Cross-eyed. These are a little harder on the eyes though.

- A Format Axes Dialog is available to change the minimum and maximum values of each variable and to modify other properties of the axes.

- The user can enter any desired viewpoint (point from which to view the plot) or focus point (point to use as the center of rotation) for the 3D graph.

- The user can control many aspects of how the surface graphs are displayed, including:
  - whether or not to display edges (the mesh framework of the surface)
  - the number of gridlines to be used
  - switch between wire mesh and filled polygons
  - view the surfaces in semi-transparent or opaque modes (Semi-transparent mode allows the user to see through the surfaces, although it is slower to rotate. Opaque mode makes the surfaces opaque and allows for much faster rotation.)
  - Surfaces can be colored using a function of three variables, \( f(x, y, z) \), or the real part of a complex function, \( f(w) \).

- A large number of intuitive control keys are available to accomplish common viewing tasks. All of these keys are specified with the corresponding feature on the menu at the top of the applet.

OTHER FEATURES INCLUDE:

- The user can print the current view along with a list of the equations/notation for the surfaces, space curves, points and vectors.

- Most discontinuous surfaces are rendered correctly.

- Level surfaces can be visualized using a special “movie mode” which recognizes when a constant, \( C \), is used in the definition of one or more functions of two variables. It asks for a start and end value, the number of steps/frames to use, and the minimum delay that should be used between frames. Then it calculates the frames (up to memory capacity) and then displays them as a movie that flows from beginning to end and then back again smoothly.

- The directional derivative can be explored visually, showing the direction vector, gradient vector, trace curve, and tangent line. The user can vary the input point \((x, y)\) and the direction of the direction vector for any surface.

- Instructors can create/save script files that can be loaded in during class as a dynamic slide show. Essentially this allows you to save a series of views with the functions and settings of the current state. This makes it easier to demonstrate more during class when time is limited.
The main goal of this project is to improve student understanding of the geometric nature of multivariable calculus concepts, i.e., to help them develop accurate geometric intuition about multivariable calculus concepts and the various relationships among them.

To accomplish this goal, the project includes four parts:

- Creating a Multivariable Calculus Visualization applet using Java and publishing it on a website: web.monroecc.edu/calcNSF
- Creating a series of focused applets that demonstrate and explore particular 3D calculus concepts in a more dedicated way.
- Developing a series of guided exploration/assessments to be used by students to explore calculus concepts visually on their own.
- Dissemination of these materials through presentations and poster sessions at math conferences and through other publications.

**Intellectual Merit:**
This project provides dynamic visualization tools that enhance the teaching and learning of multivariable calculus. The visualization applets can be used in a number of ways:

- Instructors can use them to visually demonstrate concepts and verify results during lectures.
- Students can use them to explore the concepts visually outside of class, either using a guided activity or on their own.
- Instructors can use the main applet (CalcPlot3D) to create colorful graphs for visual aids (color overheads), worksheets, notes/handouts, or tests. 3D graphs or 2D contour plots can be copied from the applet and pasted into a word processor like Microsoft Word.
- Instructors will be able to use CalcPlot3D to create lecture demonstrations containing particular functions they specify and/or guided explorations for their own students using a scripting feature that is being integrated with this applet.

The guided activities created for this project will provide a means for instructors to get their students to use these applets to actively explore and “play” with the calculus concepts.

Paul Seeburger, the Principal Investigator (PI) for this grant project, has a lot of experience developing applets to bring calculus concepts to life. He has created 100+ Java applets supporting 5 major calculus textbooks (Anton, Thomas, Varberg, Salas, Hughes-Hallett). These applets essentially make textbook figures come to life. See examples of these applets at www.monroecc.edu/wusers/pseeburger/.

**Broader Impacts:**
This project will provide reliable visualization tools for educators to use to enhance their teaching in calculus and also in various Physics/Engineering classes. It is designed to promote student exploration and discovery, providing a way to truly “see” how the concepts work in motion and living color. The applets and support materials will be published and widely disseminated through the web and conference presentations.
A series of guided explorations/assessments are being created to help students to gain a deeper understanding of the geometric nature of various concepts in multivariable calculus. Some of these explorations will also be used to help assess the effectiveness of the visualization tools and the explorations themselves at improving student understanding of various concepts. If you have ideas for future assessment topics and/or questions or functions to explore, please contact Paul Seeburger at pseeburger@monroecc.edu.

So far, the following four explorations have been completed:

- Dot Product Exploration
- Cross Product Exploration
- Velocity and Acceleration Exploration

Each of these guided explorations have been created with SurveyMonkey to make it easier to collect student response data. Students are asked a short series of Pre-Test questions before they complete the body of the exploration. A link to the applet is provided in the survey which also supplies the script for the actual exploration in the CalcPlot3D applet. After the students complete the actual exploration using the applet, for which there is a second list of questions in SurveyMonkey to complete, they are asked a series of Post-Test questions which are very similar to (or sometimes the same as) the Pre-Test questions.

As you can see below, the results have been quite positive. For example, student success on the Cross Product Exploration went from roughly 60% or less on the Pre-Test questions to 90+% on six of the Post-Test items.

Explore these exploration-assessments yourself by clicking on the Workshop/Faculty link on the left sidebar of the project website. These faculty links allow you to explore the assessments while keeping your results separate from student data.

**Invitation to Participate:**
Please consider using these materials with your calculus classes! Also recommend them to anyone who might be interested in using them, including faculty in Mathematics, Physics, or Engineering.

If you would like to participate in class-testing the materials for this project and be involved in their continued assessment and improvement, or if you simply have some comments, questions, or suggestions concerning this project, please contact Paul Seeburger at pseeburger@monroecc.edu.

To access these materials go to: [http://web.monroecc.edu/calcNSF](http://web.monroecc.edu/calcNSF)