TEACHING CONCEPTUAL UNDERSTANDING THROUGH MANIPULATIVES

AMATYC
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Our goals for this workshop

You will…

• experience the power of manipulatives
• consider using manipulatives in your classes
• know where to acquire manipulatives
• be aware of online resources
  o teacher training videos
  o student worksheets
  o virtual manipulatives
First, a CAT

Developmental math students often apply memorized algorithms and procedures without understanding the concepts.

Do you agree? If so, give an example of a topic where this occurs in your class.
Why do we use manipulatives?

• We identified a problem - students lack conceptual understanding of mathematics
  o Random application of algorithms and procedures
  o No understanding of ‘why’
  o No foundation to build on

• We saw manipulatives in our kids’ classrooms
  o Concrete objects used to make abstract concepts real
  o Why not use them in college?
Evolution of this project

- Have always used objects to illustrate math concepts
  - Calculus - ‘toys’
  - Statistics and Probability - dice, beans, cards
- Recognized a need in developmental courses
- Researched
  - Talked with elementary and middle school teachers
  - Attended NCTM workshops
Implementation

• Got grant moneys to acquire manipulatives
• Developed curriculum to integrate into our developmental classes
  • Created student manipulative activities - student worksheet with instructor page
• Shared activities with our colleagues
• Sabbatical project 2011-12
  • Expanded the worksheet packets
  • Recorded 11 teacher-training videos
  • Identified good sources of virtual manipulatives
What is “Manipulative Mathematics”?

- Student worksheet activities
  - Students use manipulatives to develop conceptual understanding
  - Leads students to discovery of concepts
  - Results in many “Aha!” moments among our students

- Teacher support
  - Training videos
  - Instructor pages
How do we use “Manipulative Mathematics”?

• Today you’ll see how we use Manipulative Mathematics activities to teach these topics:
  • Signed Numbers
  • Fractions
  • Equations
  • Slopes
Signed Numbers

• Manipulative
  o Two color counters

• Activities
  o Addition of Signed Numbers
  o Subtraction of Signed Numbers
Signed Numbers Packet

• Contents of Signed Numbers Packet
  o Instructor Page
  o Student Worksheet
  o Extra Practice

• Let’s do the Activities!
Signed Numbers Teacher Video

• All Teacher Videos include:
  • Introduction to the Manipulatives
  • How do they help students learn?
  • How can you use them?
  • Suggestions for use
  • Where to find them – physical and virtual
Signed Numbers Teacher Video
Virtual Manipulatives
Addition of Signed Numbers

nlvm.usu.edu
Color Chips – Addition
Virtual Manipulatives
Subtraction of Signed Numbers

5 - (-3)

nlvm.usu.edu
Color Chips – Subtraction
Fraction Worksheets

• Conceptual understanding
  • Naming fractions
  • Equivalent fractions
  • Fractions equivalent to one

• Fraction operations
  • Multiplication, division
  • Addition, subtraction
  • Need for the LCD
Equivalent Fractions

- **Manipulatives**
  - Fraction Tiles
  - Fraction Circles

- **Activities**
  - Fractions Equivalent to One
  - Equivalent Fractions
Equivalent Fractions Activities

• Gives students a visual image of fractions equivalent to one and to each other

• Students see how concrete (tiles) lead to abstract concepts
  • e.g., why you must multiply numerator and denominator by a common factor to make an equivalent fraction
Virtual Manipulative
Fraction Number Line

http://www.mathsisfun.com/numbers/fraction-number-line.html
Fraction Operations

• Manipulatives
  • Fraction Tiles
  • Fraction Circles

• Activities
  • Adding and Subtracting Fractions
  • Find a Common Denominator
Fraction Operations

• Students see why like denominators do not change for addition and subtraction

• Students understand the meaning of the common denominator
Fraction Operations Teacher Video
Fraction Operations Worksheets

• Model Fraction Addition
• Model Fraction Subtraction
• Model Finding the Least Common Denominator
Virtual Manipulative
Adding and Subtracting Fractions

http://nlvm.usu.edu/en/nav/frames_asid_274_g_2_t_1.html?
open=activities&hidepanel=true&from=topic_t_1.html.
Virtual Manipulative
Finding a Common Denominator
Solving Equations

• Manipulative
  • Envelopes and Counters (or paperclips)

• Activities
  • Model the Subtraction Property of Equality
  • Model the Division Property of Equality
Solving Equations Teacher Video
Solving Equations Worksheet

- To solve an equation is to discover the value of the unknown
- Students see that they must do the same operation to both sides of the equation
Virtual Manipulatives
Equations

Explorelearning ‘gizmos’
• 5 minutes free use
• Two-color counters model for 1-step equations
• Cups and counters model of 2-step equations
Explorelearning Gizmos

7 = 2x + 1 for x

7

2x + 1
Exploring Slopes

- Manipulative
  - Geoboards, rubber bands

- Activity
  - Exploring Slopes
Exploring Slopes Worksheet

Students count out the rise and the run
Exploring Slopes Teacher Video
Manipulatives at Santa Ana College

- Basic Math, Prealgebra, Elementary Algebra
- Manipulatives in the classroom
  - Class sets of manipulatives
  - Full sets in classroom where most sections meet
  - One section meeting in computer classroom
- Activities included in *Foundations of Algebra* manuscript
- Professional development workshops
What do our students say?

• My favorite was the two color counters because I have always had a hard time with negative numbers and this made me understand.
• The fraction pieces helped a lot and I was able to understand completely
• Fractions were a weakness of mine and the manipulative activity helped me improve.
What do our students say?

• Envelopes and Paper Clips - this activity made it easy and fun to understand how to solve these equations because the activity showed how many coins you had outside and from that you would get an idea as to how many were inside the envelope.

• The envelopes and paper clips helped me understand the concept of that chapter way better.

• Geoboard because I never understood slope till having it visual in my hands.
Virtual vs physical manipulatives

• Student preferences
  • Virtual 6  Physical 15  Both 2

• Why virtual manipulatives?
  • It’s easier and faster.
  • You can always go back if you still don’t get it.
Virtual vs physical manipulatives

• Why physical manipulatives?
  • It’s more interaction.
  • I remember it better if I have something solid in front of me.
  • It makes me actually think more about the problem.
  • Virtual manipulatives may cause distractions because it is on a computer and I have other interests to do on a computer rather than math.
Where can you find manipulatives?

- Teacher supply stores
- Catalogs and websites
  - National Council of Teachers of Mathematics
  - Cuisinaire
  - Learning Resources
  - ETA
  - Lakeshore Learning
- Math for Elementary School Teachers textbooks
Resources available online

Manipulative Mathematics website

- Teacher training videos
- Links to virtual manipulative websites
- Student activity worksheets – password protected (email us!)
Questions???
We’re here to help you!

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