Inspiring Mathematical Practices in Elementary Education Courses

Jennifer Earles Szydlik
University of Wisconsin Oshkosh
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A Story with a Lesson

- Eleven prospective elementary teachers

- Four basic mathematics items they couldn’t solve to start
  1) LCM(60, 105)  2) GCD (60, 105)
  3) 0.4/.05        4) Multiplication of fractions story.

- One web-based teacher resource
<table>
<thead>
<tr>
<th>Interview Tasks</th>
<th>Initial Inventory</th>
<th>Interview prior site</th>
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<th>Final Inventory</th>
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</thead>
<tbody>
<tr>
<td># 1 -- GCD (60, 105)</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td># 2 -- LCM (60, 105)</td>
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<td>1</td>
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<tr>
<td># 3 -- 0.4 /0.05</td>
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<td>0</td>
<td>5</td>
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<tr>
<td># 4 -- Fraction Mult.</td>
<td>0</td>
<td>3</td>
<td>4</td>
<td>7</td>
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</tbody>
</table>
What Happened at the Website?

• Almost no consideration of definitions.
• No mathematical exploration.
• Little attention to explanations clarifying why procedures made sense or to giving meaning to the problems.
• Little attention to notation or language.
What does it mean to practice mathematics?

Mathematical Practices include giving normative meaning and attending carefully to language, notation and definitions.

It means searching for patterns, and structures and relationships.

It means making conjectures, models and arguments.
What happened to the missing square?!
A polygon is a simple, closed curve in the plane composed only of line segments.
Suppose you are given some information about a triangle $ABC$. In which of the following cases will the information be enough to allow you to determine the exact size and shape of the triangle?

a) $AB = 8$ cm and $AC = 6$ cm and $\angle BAC = 45^\circ$

b) $AB = 8$ cm and $AC = 7$ cm and $\angle ABC = 45^\circ$

c) $\angle ABC = 75^\circ$, $\angle BCA = 80^\circ$, and $\angle CAB = 25^\circ$

$BC = 7$ cm, $AC = 8$ cm, and $AB = 9$ cm
1) Explain why it makes sense that the area of a trapezoid is always $\frac{1}{2} h (b_1 + b_2)$ where $b_1$ and $b_2$ are the lengths of the parallel bases and $h$ is the height.

You can do this by partitioning the trapezoid region into pieces, or by cutting it apart and rearranging it, or by adding on structure. You may use what you know about finding areas of rectangles, parallelograms, and triangles. See if you can find more than one way to do this.
Find a formula for the sum of the measures of the vertex angles of any $n$-gon (a polygon with $n$ sides). Make a mathematical argument that the formula you find will work for a polygon of any number of sides.
Mathematics is a World of Mental Objects

“It is impossible to distinguish and thus contrast the interpretation of the thing from the thing itself ... because the interpretation of the thing is the thing.”

Mehan & Wood
Pictures and Symbols do not Carry Meaning. Meaning Must be Negotiated.

Schipper (1982)
“Students arrive at what they know about mathematics mainly through participating in the social practice of the classroom... Students adapt their talk to the practice of their specific classroom.”

H. Bauersfeld
Much Learning is Covert

“We have come to see the constructing of mathematical meaning as kind of subjective activity in which only a few parts are under actual conscious control. All the rest seems to be available instantaneously, “at hand,” for the person and functions as orientation for action. This major part is subconsciously processed, and includes the use of language, symbols, and other means; the “framing of the situation”; how and when to do it; and so on.”

H. Bauersfeld
What Does All This Suggest for Teaching to Encourage Mathematical Thinking?

“The only way to learn mathematics is to do mathematics.”

Paul Halmos

1) Instructors must model the mathematical culture.

2) We need to make our values explicit.

3) We should negotiate meanings of problems and representations with students.

4) We should make use of “paradigm cases” to teach important lessons (like the missing square example).

What might this look like in practice?
Giving support and time to make sense of the problem
The room is silent for 5-10 minutes.
Then people begin to share ideas with their groups.
A first attempt at public solution proves inadequate. I am not the judge.
Students generated a list of things we did that were mathematical.

- Made diagrams
- Flow chart to keep track of cases
- Considered possible combinations
Don’t be afraid to leave a problem unsolved – 47 hours later we had three solutions.
I encourage students to talk – even during a presentation - as long as the talk is mathematical.
What Happens in Class?

I try to model the mathematical culture.

We all engage in mathematical thinking for at least 30 minutes.

I make features of that thinking explicit (and not just covert).

All of this is done within the context of important content.

We practice speaking and writing and listening to mathematics.
What’s the point?

Only mathematicians can model the mathematical culture for students. It is up to us to acculturate them.

“Teachers also have to be exemplary, living models of the culture wanted, with transparent modi of thinking, reflecting, and self-controlling.”

H. Bauersfeld

We want mathematically powerful elementary education students when the semester ends.
References


References

- Kaplansky, I. *Paul Halmos: Celebrating 50 Years of Mathematics*.


Jennifer E. Szydlik
szydlik@uwosh.edu