The Textbook, the Teacher, and the Derivative

Linda Leckrone
lleckron@umich.edu
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University of Michigan School of Education and Rackham Graduate School

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The Study

Background and Motivation

- Personal
- Community Colleges
- Calculus
- Derivative
Research Questions

1) How do faculty use the textbook as a resource for the planning and teaching of derivatives in first semester calculus?

2) How do teachers describe a derivative for themselves and for their students?
Methods

Five community college calculus teachers
Two classroom observations each (except online teacher)
Post-observation interviews
Formal interviews
Transcription and coding along frameworks for textbook use and derivatives.
Open coding
(Textbook analysis)
Framework

Brown – teacher tool relationship

• Offloading
• Adapting
• Improvising
• Other?

Results

How Teachers Used Their Textbooks

- Offloading
- Adapting
- Improvising
- Other textbook

Arthur
Bruce
Charles
Duncan
Edward
Other Textbook

Textbook as resource and reference
- Bruce, Charles, and Edward referred to the textbook for notation
- All participants expected students to use textbook for content help
- Charles and Edward used the textbook to define the boundaries of the calculus course

Evaluation of textbook
- “The number of problems is good. We like the layout, we like the explanations.” (Bruce)
Research Question 2

How do teachers describe a derivative for themselves and for their students?
Framework (Park)

Park’s (2013) four stages of development of student understanding:

1. a point-specific value
2. a collection of values at multiple points
3. a function
4. an operator

Derivative: \( f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h} \)

The Teachers

So, I mean, all of that is just kind of connected to me. It’s all one big circle. Slope, secant line, instantaneous rate of change, it’s all the same thing” (Charles 485-486)

First I’m going to get the verbal model [from students]. I’m going to get all three verbal models…. the instantaneous slope of the curve. It’s the instantaneous rate of change. It’s the slope of the tangent line. Right? Then I would expect [students] to give me the formal model… which is the limit as something goes to zero of the function plus something minus the function over delta x. (Duncan, 386 – 396)
The Teachers

Edward, observation 1:
• The derivative is “nothing more than a difference of functions and a limit”… (object)
• “the derivative is not distributive” (process)

Bruce, observation 1:
• $e^y = x$ is “one huge derivative” (object)
• “take the derivative of both sides” (process)
Conclusions

• The teachers used the textbook in a variety of ways.

• The teachers had (as expected) a robust understanding of conceptions of the derivative

• The teachers did not always differentiate between the derivative as a process and an object.
Future Research

- Do teacher expectations of student use of text match what students do?
- Is there a difference in how part-time vs. full time faculty use their textbook for teaching and planning?
- How do different textbooks treat the process/object layer of the derivative?
- All of the above for Integrals
Questions?

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Differentiability

Differentiable at a point:

- A function \( f \) is differentiable at a point \( c \) if
  \[
  \lim_{{h \to 0}} \frac{f(c+h)-f(c)}{h}
  \]
  exists.

Differentiable on an interval:

- Similarly, \( f \) is differentiable on an open interval \( (a, b) \) if
  \[
  \lim_{{h \to 0}} \frac{f(c+h)-f(c)}{h}
  \]
  exists for every \( c \) in \( (a, b) \)

From: http://www.sagemath.org/calctut/differentiability.html
“A function is **differentiable at** \( x \) if its derivative exists at \( x \) and is **differentiable on an open interval** \((a,b)\) if it is differentiable at every point in the interval.”

A closer look at the textbook

A function is differentiable at $x$

- if its derivative exists at $x$
- and is differentiable on an open interval $(a, b)$

if it is differentiable at every point in the interval.