How Combinatorics Sheds Light on Polynomial Roots

Randy K. Schwartz
Schoolcraft College

Abstract
Problem 19 on the October–November 2013 round of the AMATYC Student Mathematics League contest was a challenging question about polynomials, and it inspired some discussion in the Mathematics Department at Schoolcraft College, MI. As instructors, grappling with such problems is a great way to reinforce our own understanding, and that of our better students, in the important topic of polynomials, factors, and roots. Further, it shows how another branch of mathematics—combinatorics—sheds light on this topic.

19. All of the coefficients of the fourth degree polynomial \( P(x) \) are odd integers. Find the maximum possible number of rational solutions of the equation \( P(x) = 0 \).

A. 0  B. 1  C. 2  D. 3  E. 4

The author gives a further exploration of this question and addresses polynomials of degrees other than four.

Randy K. Schwartz is a mathematics professor at Schoolcraft College, a community college in Livonia, MI, where he has taught since 1984. He earned a bachelor’s degree from Dartmouth College and a master’s degree from the University of Michigan. At Schoolcraft, his teaching focuses on preparing students for careers in engineering, science, health care, and business. He is a member of the Commission on the History of Science and Technology in Islamic Societies (CHSTIS), and a member of the editorial board for Convergence, the Mathematical Association of America (MAA) online journal in the history of mathematics and its uses in teaching. In 2000, Schwartz was awarded the Democracy in Higher Education Prize (National Education Association) for his essay, “Unity in Multiplicity: Lessons from the Alhambra,” an argument for a multicultural approach in mathematics education; and in 2009 he won a Trevor Evans Award (MAA) for his article, “The Birth of the Meter,” in Math Horizons.