Why do we need an ellipsoid to represent the figure of the Earth?

This image depicts the earth’s shape without water and clouds. It looks like a sloppily peeled potato, not a smoothly shaped surface.

Calculation of geographic position on this irregular surface is very complex. A simpler reference model is needed to ease calculations.

The simplified figure of the Earth is represented by an ellipsoid of revolution.

View of the Earth from the European Remote Sensing satellite, ERS-1 from 780Km (image from DMS-NEMA presentation)

Approximated Earth Shape – Ellipsoid or Revolution

Rotate Ellipse in 3 Dimensions:

GRS80 (reference ellipsoid used in NAD83 datum)

Semi-major Axis: \( a = 6378\,137.0\,\text{m} \)
Semi-minor Axis: \( b = 6\,356\,752.314\,140\,\text{m} \)
Flattening Ratio: \( f = (a-b)/a = 1/298.257\,222\,101 \)
First Eccentricity squared: \( e^2 = 2f - f^2 = 0.006\,694\,380\,022\,901 \)

These parameters are derived from an RMS best fit to the Geoid.
Latitute and Longitude

Latitude
Parallel concentric circles starting at the equator, decrementing in radius towards the poles.
Time & Longitude

**Zero Hour at prime meridian**
Modern datums reference the **Greenwich Meridian** as the prime meridian through the Royal Greenwich Observatory, Greenwich England.

Longitudinal lines converge at the poles.

Above: View of the earth, looking down at the north pole.

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**Horizontal Positioning (astronomic methods)**

- 1) Right Ascension & Declination
  
  Used for star catalog

- 2) Hour Angle & Declination

- 3) Azimuth & Altitude
  
  Used for field observations
Earth-Center Earth-Fixed (ECEF) coordinates

Ellipsoid
(e.g., GRS-80, WGS-84)

Point #1, Arizona
Coordinates:
(–X₁, –Y₁, +Z₁)
(φ₁, λ₁, h₁)

–X axis
(180°W)

Equatorial plane

–Y axis (90°W)

–Z axis

+Y axis (90°E)

+X axis (Prime meridian)

Geoid
("mean sea level")

Earth Mass Center

+Z axis (parallel to axis of rotation)
NORTH

Geographic: Reference to rotation point of earth, and running with lines of longitude.

• Geodetic: Either direct observed through geodetic tools such as GPS/GNSS or computed from Astronomic observation with one or many correction terms.

• Astronomic: Derived from direct optical measurement/observation to celestial objects.

• True: Legal north. As described by BLM, this would probably be a form of astronomic north.

Grid: Reference to mapping projection. Only related to a line of longitude at one point/line typically.

Magnetic: North as indicated by a compass.

Record: What is shown on an historic document. May or may not be a derivative of one of the above.

Assumed: Randomly assigned.

Latitude, Longitude & Time

Latitude: Altitude measured to a cataloged star
Longitude: must know time to determine

Time:
- Sidereal: Hour angle from first point of Aries
- GMT: Time at Greenwich
- UTC: Atomic time
- GPS time: Atomic time used with GPS satellites

Terms:
- Aberration: Movement of light from a star, relative to the motion of the earth
- Nutation: Oscillation of earth’s axis, 18.6 year cycle
- Precession: Motion about the equator
- Polar Motion: Motion about the pole, 434 day period
Networks: Systems of measurements tied together to form the backbone of maps

Convergence: North on the ‘grid’ vs. ‘true north’

Geodesic lines: the appearance of a straight line on the ground to a curved line on a map

Spherical Excess: $180 + e$

Small amounts of excess in angle on large triangles on the surface of the earth

Vertical Position

Differential (spirit) Leveling

Trig Leveling

ALSO:
- GPS/GNSS heights
- Gravity measurements
- Barometric Leveling
- Sounding
Because the figure of the Earth is not a sphere, the radius of curvature changes with Latitude

Oblate $s_1 > s_2$

the Greek letter ‘phi’ $\varphi$ is used to express latitude.
Geometric Mean Radius of Curvature ($R_G$)

$R_G$ is essentially the ‘average’ radius of curvature at a point on the ellipsoid, and is the one we will be using in future calculations. It can be expressed as a function of the Meridian Radius ($R_M$) and the Prime Vertical Radius ($R_N$).

\[
R_M = \frac{a(1-e^2)}{(1-e^2 \sin^2 \varphi)^{3/2}} \quad \text{and} \quad R_N = \frac{a}{(1-e^2 \sin^2 \varphi)^{1/2}}
\]

\[
R_G = (R_M R_N)^{1/2} = \frac{a(1-e^2)^{1/2}}{\sin^2 \varphi} = \frac{b}{1-e^2 \sin^2 \varphi}
\]

This is the formula we will be using in our exercise.

$\varphi = \text{geodetic latitude of point}$

\[
R_G = \frac{b}{1-e^2 \sin^2 \varphi}
\]
Geoid

Surface of constant gravitational equipotential (a level surface) that best corresponds to global mean sea level. Used as a reference surface for modern vertical datums, such as NAVD 88.

The equipotential surface of the Earth's gravity field which best fits, in a least squares sense, global mean sea level. Even though we (NGS) adopt a definition, that does not mean we are perfect in the realization of that definition. For example, altimetry is often used to define "mean sea level" in the oceans, but altimetry is not global (missing the near polar regions). As such, the fit between "global" mean sea level and the geoid is not entirely confirmable. Also, there may be non-periodic changes in sea level (like a persistent rise in sea level, for example). If so, then "mean sea level" changes in time, and therefore the geoid should also change in time. These are just a few examples of the difficulty in defining "the geoid".

{http://www.ngs.noaa.gov/GEOID/geoid_def.html}

Each epoch of the Geoid is a solid, nonmoving surface.

Potential Energy on the Geoid

The gravity vector is perpendicular to all points on the geoid surface, and the geopotential number (acceleration of gravity) is equal at all points. The surface is at equal potential, or simply, a level surface (like water that is not flowing).

If you were rowing a boat along the geoid, it would take the same amount of force to row \textit{uphill} as it would to row \textit{downhill}.

\textit{Geoid slope is a slope with no slope}

The geoid is only \textit{‘sloped’} when compared to the ellipsoid.
** Distance along plumb line is a curved line **

A line is *normal* to a curve if it is perpendicular to a tangent line to that curve at the point where it intersects the curve (aka: instantaneous slope of the curve, the calculus first derivative)
EGM96 Geoid contours

When compared to the *smooth* surface of the ellipsoid, the geoid appears to be sloped.

These slopes result in height differences that can be mapped as a contour map.

The contours represent differences in height between ellipsoidal height and orthometric height.

Geoid Model

Hybrid geoid file that will model the arithmetic difference between measured NAD83 ellipsoid heights and NAVD88 orthometric height at a measured NAD83 latitude, longitude, and ellipsoid height (h); modeled geoid height (separation) = N

Modeled Orthometric Height (H):

\[ H \approx h - N \]

Examples of *Hybrid geoid* files from NGS: Geoid03, Geoid99, Geoid96

*Geoid* files are used when surveying with GPS
Geoid Model Orientation to Job Site

Geoid separation (N) is a direct function of observed NAD83 lat/lon/height. If the project is not properly georeferenced, error will occur. How much error? We will examine that later in this class.

Be careful also of data collector default settings. Notice that the 1/f that defaults here in TDS is neither GRS80, nor WGS84, but a +/- average of the two. How bad is this? We will examine that later in this class.
Orthometric Height Calculation

\[ H \approx h - N \]

- \( h \) (Ellipsoid Height) = Distance along ellipsoid normal (Q to P)
- \( N \) (Geoid Height) = Distance along ellipsoid normal (Q to \( P_0 \))
- \( H \) (Orthometric Height) = Distance along Plumb line (\( P_0 \) to P)

Height Relationships in Arizona
NOAA Manual NOS NGS 5

State Plane Coordinate System of 1983

James E. Swain

National Geodetic Survey
Rockville, MD
January 1989

Reprinted with minor corrections
March 1990

Reprinted February 1991

Reprinted July 1992
Reprinted January 1993
Reprinted August 1995
Reprinted April 1994
Reprinted January 1995
Reprinted September 1995

http://geodesy.noaa.gov/PUBS_LIB/pub_index.html

FREE GOVERNMENT PUBLICATION!!

Every surveyor who uses GPS should have a copy of this manual.
Map Projections

Map projections are used to go from positions on the ‘round globe’ (ellipsoid) to the ‘flat’ surface on a map.

Map Projection =
Ellipsoid → Mapping Plane
Map Projection Types

Three basic types of projection:

- Plane
- Cone
- Cylinder

A cylindrical map projection touching at the equator is named after Mercator

Map Projection Types

There are several hundred types of projections. Some of the more common for surveying are:

- **Mercator**: Cylinder touches at the equator
- **Transverse Mercator**: Cylinder touches at a meridian
- **Lambert**: Conical touching at a parallel
- **Oblique Plane**: Plane touching at one point only locally oriented to a project.
Projecting a Map

The ‘projection’ is like a shadow cast through the globe onto a piece of paper
Projected from an Ellipsoid to a Mapping Plane

Map Distortion

All projections have distortion. The further the map surface from the point/line of contact to the globe, the greater the distortion.

DISTORTION is not ERROR
Surface Development

Cylinder and Cone projections can be ‘developed’ into a flat map by cutting them and laying them out flat.

The result is a flat or planer (cartesian) surface.

- Plane surveying on a planer surface

Plane projections do not need to be developed

Mapping Plane Orientation

A mapping projection may be oriented to an ellipsoid in several ways:

- Tangent: Touching at one point only
- Secant: crossing through the reference ellipsoid, touching at more than one point
- Not touching the reference ellipsoid at all (above, or closer to the surface of the earth)
Map projections used in AZ

- **Arizona Coordinate System**
  
  *commonly referred to as state plane coordinates (SPC)*

- **Universal Transverse Mercator** (UTM) coordinates

- City coordinate systems:
  
  Scottsdale, Tempe, Chandler, etc…

- On the job coordinates (aka: local coordinates)
  
  Northing = 10,000
  
  Easting = 10,000

*** There may be many different horizontal datums and different adjustments referenced in all of these systems!!

ARS 33-131 State Plane Coordinates Zones

33-131. Arizona coordinate system, 1983; zones; composition

A. The Arizona coordinate system, 1983, is the system of plane coordinates which has been established by the national geodetic survey for defining and stating the positions or locations of points on the surface of the earth in this state.

B. The Arizona coordinate system, 1983, contains three zones as follows:

1. The **west zone**, composed of La Paz, Mohave and Yuma counties.
2. The **central zone**, composed of Coconino, Maricopa, Pima, Pinal, Santa Cruz and Yavapai counties.
3. The **east zone**, composed of Apache, Cochise, Gila, Graham, Greenlee and Navajo counties.

C. In any land description in which the Arizona coordinate system, 1983, is utilized the system shall be designated "Arizona coordinate system, 1983, ________ zone", with the name of the appropriate zone inserted.

http://www.azleg.state.az.us/ArizonaRevisedStatutes.asp?Title=33
ARS 33-132 State Plane Coordinates

33-132. Coordinates of system; zone definitions

A. The plane coordinates of a point on the earth's surface, to be used in the position or location of such point in the appropriate zone of the system, shall consist of two distances, expressed in feet and decimals of a foot (foot value 0.3048 meter exact). One of these distances, to be known as the "X-coordinate", shall give the position in an east-and-west direction, and the other, to be known as the "Y-coordinate", shall give the position in a north-and-south direction. These coordinates shall depend on and conform to the coordinates on the Arizona coordinate system, 1983, of the horizontal control stations of the national geodetic survey in this state, as these coordinates have been determined by the survey.
B. For the purpose of more precisely defining the Arizona coordinate system, the following definitions of the national geodetic survey are adopted:

2. The Arizona coordinate system, 1983, central zone, is a transverse mercator projection of the North American datum, 1983, having a central meridian 111° 55' 00" west of Greenwich, on which meridian the scale is set at one part in ten thousand too small. The origin of the coordinates is at the intersection of the meridian 111° 55' 00" west of Greenwich and the parallel of 31° 00' 00" north latitude. This origin is given the coordinates of "X" equals seven hundred thousand feet and "Y" equals zero feet.

SPC Zones – CM’s

Zone Defined by:
- Central Meridian (CM)
- Scale at CM
- False easting at CM
- False northing at a base latitude
- Geodetic Datum
- Ground Control
- Linear units used
SPC Map Projection Orientation

AZ SPC map projections are *secant* transverse Mercator projections.

SPC Scale Factors

Scale Factor: distortion from the ellipsoid to the mapping plane.

In a SPC zone, the scale factor distortion has been minimized throughout the width of the zone.
SPC

Cross Section of a Semicicricle Projection

Round Earth - Flat Map

Flat Earth - Round Map

- Ellipsoid
- Map Projection Plane
- Distortion
Many Projections in one view
112.55 looking west

112.55 Degrees West Longitude
Looking West
Universal Transverse Mercator (UTM)

The Universal Transverse Mercator Coordinate (UTM) system provides coordinates on a world wide flat grid for easy computation.

The UTM Coordinate system divides the World into 60 zones, each being 6 degrees longitude wide, and extending from 80 degrees south latitude to 84 degrees north latitude.

The polar regions are excluded. The first zone starts at the International Date Line (longitude 180 degrees) proceeding eastward.
Datum Realization

Passive Control

NAD27 → NGVD29
NADCON (27-86) → VERTCON
NAD83(86) → NAVD88
NADCON (86-HARN)

Active Control

NAD83(HARN) (92, 99, etc.) → NAD83(CORS96:2002.0000)
NAD83(2007) → Multi Year CORS
**Orthonomic heights above NAVD 88 Datum**

Prototype orthometric heights are now being made available as a precursor to the completion of NAVD-92 and the replacement of NAVD 88 with a new geopotential reference system. The following height reflects the current best estimate of the true orthometric height, based on the existing gravimetric geoid model. This height is subject to change as data and modeling for the gravimetric geoid change throughout the lifetime of the NAVD-92 project, or as new realizations of the TRF are adopted. However, at the completion of NAVD-92, these heights will supersede the NAVD 88 heights throughout the United States.

**Approximate Orthometric Height:**
- NAVD 88 Height: 408.728 meters
- Datum Shift (NAV88 minus NAVD 29): 0.665 meter

**Converted to NAVD 29 Height:**
- 408.063 meters
OPUS: Online Positioning User Service

Upload your data file.

To transfer your GPS observation to the National Spatial Reference System.

What is OPUS? FAQs

You selected 2011 frame for processing your observation.

olish6@cap-lx.com

* Email address - your solution will be sent here.

* Data file of dual-frequency GPS observations. sample

PHASE 3 GPS

Mode Integrated Antenna (RA) Model 2

Antenna type - choosing wrong may degrade your accuracy.

2.000 meters above your mark.

Antenna height of your antenna's reference point.

Options to customize your solution.

Upload to Rapid Static for data > 15 min. < 2 hrs.

Upload to Static for data > 2 hrs. < 48 hrs.

Options to customize your solution.

Solution formats

Add details to your report
standard solution

Base stations

Type in 4-char site ID, or select from map, any CORS you wish to explicitly include or exclude from your solution
Sample

State plane coordinates

Overrule your native SPGZ zone
AZ (0202) Arizona Central

Geoid Model

Customize your orthometric height model
GEOID03

Contribute to a project

Enter the project identifier provided by your project manager.
CAP-11AA

My profile

Customize OPUS defaults for future solutions
Set/Reset my profile

Publish my solution

Share your solutions
Yes, publish

Upload to Rapid-Static for data > 15 min. < 2 hrs.

Upload to Static for data > 2 hrs. < 48 hrs.

* required fields.

We may use your data for internal evaluations of OPUS use, accuracy, or related research.

Website Owner: National Geodetic Survey / Last modified by NGS.OPUS Tuesday, 24-Jan-2012 09:30:00 EST
Good Luck On Your Test!