Which side to put the survival blanket?
Analysis and suggestions for activities with students

Laurence Viennot and Nicolas Décamp

Abstract

Although a survival blanket is a well-known object, it is not obvious which side, silver or gold, should be put outside in order better to protect against cold. This paper sums up the reasons why this problem may be instructive, beyond its practical implications: Common ideas and instructions for use share a restricted analysis, limited to the high reflective power of the silver side. An analysis and a solution of this problem are proposed, and some activities with students are suggested.

Why this topic?

A practical context

A survival blanket is not of everyday usage but it is a well known device that any careful hiker or alpinist should have in his/her mountain sack. Most often, it is made of a very thin film of mylar with one side silvered, very shiny, and the other of gold colour. In case of an emergency, namely a need to protect against cold, a question presents itself: which side should be outside?

Infra red and emissivity on stage

From a conceptual viewpoint, this topic is interesting in that it puts into play a property of matter: emitting radiation due to its non zero temperature. In the range of temperatures at stake in situations considered here, infrared radiation is dominant: At about 300 K the maximum spectral density emitted by matter is at about 10 µm. Moreover, the state of surface of a given body introduces a coefficient – called “emissivity” – by which the values characterizing what is called the “black body radiation” has to be multiplied to find the power actually radiated. This coefficient may depend strongly on wavelength (Besson 2009).

The topic of the emergency blanket is an opportunity to introduce these concepts and construct a line of reasoning that makes use of them.

A non obvious answer

Besides intellectual benefits, this topic is interesting because the question posed in the title of this paper is not convincingly answered in popularization literature nor in the instructions for use found on the emergency blankets in sold.

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Common views

**Students’ ideas**

A first striking feature in students’ lines of reasoning is the idea that “silver” (the silvered side) reflects light better than “gold” (the golden side), a correct idea. Starting from this, they quasi unanimously (Viennot & Decamp, in progress: 20 PhD students, 7 prospective teachers fourth year at university) conclude that the silver side should, in case of hypothermia, be turned inwards “to reflect heat” or to “redirect heat toward interior”, or “to retain heat”.

Three remarks can be made concerning such a line of reasoning:

► It only mentions a radiant process.
► It does not examine the constantly increasing quantity of stored energy that this analysis would imply in case of steady production of “heat” (for instance due to the victim’s metabolism).
► It is exclusively focused on the space between the body of the victim and the survival blanket, thus ignoring whatever may happen between the blanket and the exterior. Turned outside, the silver side would emit less radiant power than the gold side (see Appendices 1 and 2): Wouldn’t this be a favorable case as well?

We recognize some questions that may be posed when students analyse green house effect by saying, for instance: “There is more energy coming in than going out” (second remark) or reasoning (third remark) without taking into account any retroaction from “downstream” (the exterior) on “upstream” (the interior). Briefly put, the line of reasoning students commonly adopt consists in following the radiation emitted by the warmest source and deduce thermal effects from possible reflections or absorption on the path of this radiation, in a linear causal style: a series of “one cause-one effect” links with a chronology surreptitiously suggested, which does not take into account the whole system at a time.

The first and second remarks underline the need for drawing attention to a mode of transfer by conduction and convection, in parallel with the radiant transfer. The third one makes it necessary to identify the relevant parts of the whole system in play.

**Popularisation and instructions for use**

This reasoning focused on reflection seems to be also the dominant argument underlying current explanations and instructions for use. Most frequently, it is said that to protect against an external source of heat (for example the Sun or a fire) we should turn the silver side outwards whereas to avoid hypothermia, we should do the reverse.

The encyclopedia Wikipedia does not even mention which side to put inside, but it strongly suggests that this should be the reflecting one (i.e. silver):

The theory behind thermal blankets is that heat is redirected back toward the object emitting the heat. [http://www.ehow.com/info_8574832_thermal-blanket.html](http://www.ehow.com/info_8574832_thermal-blanket.html)

It is extremely common to find on the internet explanations like the following one:

People lose heat through thermal radiation. All objects radiate infrared energy. The warmer the object, the more energy is radiated, cooling the object. A thermal blanket is more than 80 percent reflective. That means that more than 80 percent of the thermal energy that reaches it is deflected back towards its source. When someone is wrapped in a thermal blanket, his own reflected infrared heat is reflected back towards him, warming him up more quickly. Read more: [http://www.ehow.com/how-does_5145153_thermal-blanket-work.html](http://www.ehow.com/how-does_5145153_thermal-blanket-work.html)
Content analysis

At this step it is useful to reexamine the basic concepts of transfer by radiation and surface properties, recalled in Appendices 1 and 2. The reader will find hereafter a quick access to a first solution that will at least give an idea of the relevant variables and parameters. We are not suggesting, far from it, that the most appropriate pathway for students would be necessarily this direct conceptual trajectory.

A simple model of the system

We can see the system as composed of

► An object, or body, O at temperature $T_o$ considered as uniform – i.e. the same at any point at a given time. This relies on the hypothesis that the time scale of the temperature decay is large with respect to the time scale of temperature transfer inside the body (Vollmer 2009). In other words, the conductivity inside the body is very high, compared to the other conductivities in play.

► A layer between the object and the emergency blanket, in which two kinds of transfer may occur:
  • Conductive/convective: “CC”
  • Radiant

► The emergency blanket, at temperature $T_b$ (The two sides of the blanket are at same temperature, due to its very small thickness).

► The external boundary layer, in which two kinds of transfer may occur:
  • Conductive/convective: “CC”
  • Radiant

► The external air, at temperature $T_e$. No other external source of energy (for instance the Sun) is considered here.

Using a plane geometry, we might represent this system with a diagram like in Figure 1.

![Diagram of the system](image)

Figure 1. A schematic diagram of the structure of the “emergency blanket” system: object or body, emergency blanket and external air, and, in light grey, two interfacial zones (including boundary layer of external air). Each body in dark grey is considered at uniform temperature. The simplistic plane geometry is just to facilitate the representation.

In what follows, concerning the object or body O, we will only represent their interfaces with the transfer zones.
**A way to pose the problem**

The external temperature is $T_e$. A constant power $\Phi$ is transformed (metabolism of a living being, or a lighting bulb) in an object or body which is wrapped in the blanket. We wait for stationary state. In order to protect against hypothermia, we need to maximize $T_o$, hence $T_o-T_e$. Is the final temperature of the object, $T_o$, larger with silver outward ($T_{os}$) or gold outside ($T_{og}$)?

In the next section, we will also consider another way to pose the problem: given $T_o$ and $T_e$, how to minimize the flux $\Phi$ which travels through the system?

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**A simple model of the transfers: linear analysis**

The approximation consisting in expressing the net radiant flux between two bodies as a linear function of their temperature difference has been extensively discussed by Vollmer (2009, see Appendix 3). Let us assume that we can appropriately implement such a linear treatment. Concerning the conductive-convective flux, a linear treatment is commonly accepted (Brau 2006).

Let
- $C'$, $C$ the coefficients of net thermal transfer via conduction and convection (CC), respectively in the upstream and downstream part of the system;
- $A'$, $A$ the coefficients of net radiant transfer respectively upstream and downstream the blanket. (For each side of the blanket concerned, silver or gold, $A_s$, $A_g$, respectively).

All these coefficients depend in particular of shape factors.
Due to stationary state, it is
\[ \Phi = (C'+A')(T_o - T_b) = (C+A)(T_b - T_e) \]  \hspace{1cm} (1)

Local implications

For silver outside
\[ \Phi = (C+A_s)(T_b - T_e), \]
and
\[ T_{bs} - T_e = \frac{\Phi}{C+A_s} \] \hspace{1cm} (2)

For gold outside
\[ \Phi = (C+A_g)(T_b - T_e), \]
and
\[ T_{bg} - T_e = \frac{\Phi}{C+A_g} \] \hspace{1cm} (3)

From equations (2) and (4) – see also Figure 3a – we deduce that, given \( \Phi, T_e, C, A_s \) and \( A_g \), the temperature of the blanket, \( T_b \), depends only on which side of the blanket is put outside.

Given that \( A_s < A_g \), we have \( T_{bs} > T_{bg} \), which means that putting silver outside will increase \( T_b \) with respect to the case when gold is outside. The difference \( T_{bs} - T_{bg} \) will be all the larger as \( C \) is small (dry and calm weather).

In case of high value of \( C \), due to wind and/or rain, then we have
\[ C >> A_s, A_g, \]
then \( T_{bs} \sim T_{bg} \sim T_e \)

The temperature of the blanket is practically that of the external air, whatever side is put outside.

Similarly, from equations (3) and (5) – see also Figure 3b – we deduce that given \( \Phi, T_b, C', A_s' \) and \( A_g' \), the temperature of the body, \( T_o \), depends only on which side of the blanket is put inside. The value of \( T_o - T_b \) will be all the larger as \( C'+A' \) will be small. Given \( C' \), \( T_o - T_b \) will be the largest with \( A' \) as small as possible, that is with silver inside. The difference \( T_o - T_b \) will be all the larger as \( C' \) is small (air layer with dry and calm conditions between body and blanket).
The blanket has only one silver side, it is therefore impossible to maximise at the same time $T_o-T_b$ and $T_b-T_e$.

But what we seek to maximise, given $\Phi$, $T_e$, $C$, $C'$, $A_s$, $A_g$, $A'_s$, $A'_g$ is not the temperature of the blanket, nor the difference $T_o-T_b$, but the temperature of the body, or, equivalently, the value of $T_o-T_e$.

To solve this problem, we have to analyse the whole system.

*Treatment for the whole system*

Let $G$ defined by $\Phi = G(T_o-T_e)$

It is

$$\frac{1}{G} = \frac{1}{C' + A'} + \frac{1}{C + A}$$

To have the highest possible value of $T_o-T_e$, given $\Phi$, we need to minimize $G$, given $C$, $C'$, $A$, $A'$, by choosing which side to put outside (silver or gold). That is to say, we have to compare

$$\frac{1}{G_s} = \frac{1}{C' + A'_s} + \frac{1}{C + A_s}$$

(gold outside) and

$$\frac{1}{G_g} = \frac{1}{C' + A'_g} + \frac{1}{C + A_g}$$

(silver outside)

Each of these expressions might be seen as the “total resistance” for the energy transfer from object to external air in each situation.

The largest value designates the most favourable case to protect the body against cold.

We now see that, provided we consider a quasi-static situation, namely with the same power flux along the system at a given time, two possible formulations of the problem lead to the same solution:

In case of steady state with $\Phi$ constant in time, the temperature of the body, $T_o$, has the highest possible value when $G$ is minimum.

In case of given temperature of the body, $T_o$, the minimum value of $\Phi$ is when $G$ is minimum.

*How to find the most favourable case?*

In case we might make a supplementary hypothesis, that is consider identical shape factors for the emission of a given side be it downstream and upstream (certainly the most contestable hypothesis in this simple model), simple calculations may give an idea of the best compromise.

► If the shape factors are taken as equal, it is $A'_s = A_s$ and $A'_g = A_g$, then we have to compare the “total resistances”

$$\frac{1}{G_g} = \frac{1}{C' + A'_s} + \frac{1}{C + A_s}$$

(gold outside) and

$$\frac{1}{G_s} = \frac{1}{C' + A'_g} + \frac{1}{C + A_g}$$

(silver outside)

► If $C = C'$, the two expressions take the same value, whatever $A_s$ and $A_g$, and the two situations are equivalent to protect against cold.

► In order to compare the “total resistances” in the two situations, we can compare the “total conductances”, $G_g$ and $G_s$ as well. The best compromise is the one corresponding to the lowest value of $G$. Hence we have to calculate:

$$G_s - G_g = \frac{(C' + A_g)(C + A_s)}{C' + A_g + C + A_s} - \frac{(C' + A_s)(C + A_g)}{C' + A_s + C + A_g}$$
As said above, if $C = C'$, both situations (gold outside or silver outside) are equivalent to protect against cold whatever $A_s$ and $A_g$.

If not, given that $A_g > A_s$, the sign of $G_s - G_g$ is the same as the sign of $C - C'$.

In practice, in case of conditions favouring a high conducting-convective thermal transfer outside the blanket ($C' < C$, wind and rain, $G_g < G_s$), the gold side of the blanket should be put outside, and in the opposite case ($C' > C$, dry and calm weather) the silver side should be put outside.

**An electrical analogy**

The linear model which served in the previous section strongly reminds us of the case of electric circuits. Still considering the shape factors as identical upstream and downstream, it is possible to re-write the linear treatment presented above with notations and words used in electricity.

Let $C' = 1/R'_C$, $C = 1/R_C$ the conductances of two resistors, respectively upstream and downstream (following the flux of charge) point B (blanket); $R'_C$, $R_C$ are corresponding resistances.

$A' = 1/R'_A$, $A = 1/R_A$ the conductances respectively upstream and downstream point B (for each side of the blanket concerned, silver or gold, $A_s$, $A_g$ respectively) of two other resistors put in parallel of the two first ones.

$\Phi$ the flux through this compound resistor (Fig. 4).

In a quasi stationary regime, it is:

$$\Phi = (1/R'_C + 1/R'_A)(To - Tb) = (1/R_C + 1/R_A)(Tb - Te)$$

or, continuing with the electrical analogy, and just changing the notations for the flux and the temperatures

$$I = (1/R'_C + 1/R'_A)(Vo - Vb) = (1/R_C + 1/R_A)(Vb - Ve)$$

Let $G$ be such that $\Phi = G(To - Te)$, or with new notations $I = G(Vo - Ve)$; $G$ is a total conductance. The total, or “equivalent”, resistance is $1/G$.

It is

$$\frac{1}{G} = \frac{1}{1/R'_C + 1/R'_A} + \frac{1}{1/R_C + 1/R_A}$$

![Diagram](Figure 4. A way to represent the system between body and external air)
The largest possible value of $To–Te$, given $\Phi$, and the smallest possible value of $\Phi$, given $(To–Te)$ correspond to the smallest possible value of $G$, given $C, C', A, A'$. 

One has to compare:

$$\frac{1}{1/R'_{C} + 1/R'_{As}} + \frac{1}{1/R_{C} + 1/R_{Ag}} \quad \text{(gold out)}$$

and

$$\frac{1}{1/R'_{C} + 1/R'_{Ag}} + \frac{1}{1/R_{C} + 1/R_{As}} \quad \text{(silver out)}.$$ 

The larger value designates the case which is the most favourable to protect against cold. The resistor which is equivalent to the system between body and external air is composed of two resistors in series, each of which comprises two resistances in parallel. We want to maximise the equivalent resistance of this compound resistor (remember that one can replace $\Phi$ by $I$ and $To–Te$ by $\Delta V = Vo–Ve$).

As previously we take $R'_{As} = R_{As}$ and $R'_{Ag} = R_{Ag}$.

Hence, changing the external side of the blanket – gold or silver – comes down to exchanging the two radiant "resistors".

One may, for instance, compare the time constant for a discharging capacitor in the following case (Figure 5): A capacitor of 10 F is charged (1.2 Volt) then discharged in a series of two resistors, each one consisting of two resistances in parallel. Cases compared may be, for instance: two 100 $\Omega$ resistors in parallel, in series with two 1 $\Omega$ resistors in parallel, versus 100 $\Omega$ in parallel with 1 $\Omega$, in series with 100 $\Omega$ in parallel with 1 $\Omega$.

![Image of electrical equipment](image)

**Figure 5.** Material used to compare the time constants of a discharging capacitor depending on the arrangement of four resistances.

It is striking, although not surprising, to see that the capacitor discharges much more slowly when the large resistances are in parallel and the small resistances also in parallel, compared to the case when each large resistance is in parallel with a small one. Putting small resistances in parallel each one with a large resistance comes down to short circuit both large resistances. This idea of “short circuit” provides students with a good way to remember the rule to be applied in case of a risk of hypothermia: put larger resistances to heat transfer together, that is silver outside with a dry and calm weather, silver inside with rainy and windy weather.
Possible activities with students

Given the complexity of the topic, an appropriate target population might be prospective teachers or students of upper division courses at university. However, some of these activities may turn out to be quite accessible to younger students, depending on the context. We just speak of “students” in the following.

A first possible use of this topic with students might be to favour discussions, before any calculation.

A possible starting point is to ask about what they would do to protect against hypothermia, knowing that – see above – everybody answers he/she would put silver inside.

It may then be pointed out that total or even partial reflection of “the heat” toward the body poses a question of divergence: will the temperature increase indefinitely, in case of continuous power input – an approximation for human metabolism? Here the target may be to make necessary the idea of another type of power transfer, besides the radiant one: transfer by conduction and/or convection.

Depending on the available time, a model like in Figures 1 and 2 may be provided to and discussed with, or constructed with/by, students.

Students may be asked their preference about, and/or asked to construct, a statement of the problem in terms of physical quantities: temperature to be maximised or flux to be minimised.

A series of sentences can be submitted to students who would have to approve/disapprove with arguments, for instance considering a steady state with constant power emitted by the body:

► The flux emitted by the external side of the blanket is always the same, \( \Phi \).
► For a given external temperature \( Te \), and convective conditions, the outward CC-flux depends only on the temperature of the blanket.
► For given convective conditions, increased CC-flux means larger temperature difference, here \( Tb - Te \).
► The radiant flux out of a surface depends on this surface’s temperature and emissivity (high for gold and low for silver).
► For a given total flux, \( \Phi \), and given conductive-convective conditions, the proportion of emitted power under “CC” or radiant version depends on the temperature and emissivity of the external side of the blanket.
► Given \( \Phi \) and \( Te \), each side of the blanket will have a given temperature when put outside (\( Tb_s \): silver outside; \( Tb_g \): gold outside), depending on its emissivity: more net radiant power emitted, less CC-flux (gold); less net radiant power emitted, more CC-flux (silver), with the same total \( \Phi \) transferred in both cases.
► Given \( \Phi \) and \( Te \), there will be less radiant power emitted with silver outside than with gold outside, therefore more power transmitted by CC-transfer. More CC-transfer means larger temperature difference, as stated afore.

Therefore \( Tb_s \) (silver outside) > \( Tb_g \) (gold outside)

► A way to increase \( T_o - Tb \), for given convective conditions between object and blanket, is to reduce the net radiant transfer from object to blanket.
► A high rate of reflection – on the blanket – of the power emitted by the body toward this body (a kind of “green house effect”, as students say) would contribute to reduce the net
radiant transfer from body to blanket, therefore to increase the CC-transfer, hence $T_0 - T_b$. Such is the case when silver is inside.

To sum up: putting silver outside increases the temperature difference between air and blanket, but putting silver inside increases the temperature difference between blanket and object. What is the better compromise?

Beyond this point, there is no obvious way to progress without an algebraic model and some calculation.

To this end, the previous section can be used. It makes it possible to confront this model with the students’ responses given, or not, about the sentences just listed.

**The electrical analogy**

The electrical analogy lends itself both to a practical activity with convincing effects concerning the survival blanket and to revisiting electric circuits. Paradoxically enough, it happened during interviews that a student declared: “I have better understood resistances in parallel and short circuits.”

**Some experiments at home**

Of course the activity that seems the most natural to propose to students is to perform an experiment with the same body wrapped with the emergency blanket material, silver outside V/s gold outside. Some results of home-made experiments with hot water in a freezer ($-17 \, ^\circ C$) and two sets of values for the volume of water, the initial temperature and the cooling time, are given in Tables 3 and 4. The very simple material used is shown in Figure 6.

Given that the inside of the freezer can be seen as a calm and dry environment, the results confirm the previous analysis: less cooling with silver outside. However the differences in final temperature are not very large compared to the fluctuations between the results found when replicating a given experiment. During a short teaching session, it may be difficult to take all the time that is necessary to reach a safe conclusion, and this topic might be more appropriate to a student project.

<table>
<thead>
<tr>
<th>“Gold” outside</th>
<th>“Silver” outside</th>
</tr>
</thead>
<tbody>
<tr>
<td>12.5 °C</td>
<td>15 °C</td>
</tr>
<tr>
<td>13 °C</td>
<td>16 °C</td>
</tr>
<tr>
<td>12 °C</td>
<td>15 °C</td>
</tr>
<tr>
<td>13 °C</td>
<td>16 °C</td>
</tr>
</tbody>
</table>

Table 3. Final temperature of 210 cm$^3$ of water in a plastic bowl, with some air above, in a bag made of emergency blanket, gold or silver outside, starting from 38 °C, after an hour in a freezer at $-17 \, ^\circ C$. The experiment was performed four times, see each line of the Table.

<table>
<thead>
<tr>
<th>“Gold” outside</th>
<th>“Silver” outside</th>
</tr>
</thead>
<tbody>
<tr>
<td>39 °C</td>
<td>44 °C</td>
</tr>
<tr>
<td>40 °C</td>
<td>43 °C</td>
</tr>
<tr>
<td>38 °C</td>
<td>43 °C</td>
</tr>
<tr>
<td>39 °C</td>
<td>44 °C</td>
</tr>
</tbody>
</table>

Table 4. Final temperature of 65 cm$^3$ of boiling water, filling up a plastic box, covered with emergency blanket, gold or silver outside, after 20 min in a freezer at $-17 \, ^\circ C$. The experiment was performed four times, see each line of the Table.
As for the case of a windy and rainy weather, what we obtained two times in a row during a stormy day, with \(65 \text{ cm}^3\) of boiling water, filling up a plastic box, covered with emergency blanket, gold or silver outside, after 15 min on a balcony, was the same final temperature: 38.4 °C with the first experiment and 38.1 °C with the second one. This fits the case of very high convection-conduction coefficients analysed above.

This said, even if the preceding results are consistent with the previous analysis, home experiments seem insufficient to explore all the possible cases, particularly the one with high external conduction.

Note: An experiment performed in the frame of a scientific association “Planète Sciences”, backed up by the popularisation service of CNES (National Center of Spatial Studies in France) can be found, in French, at http://archive.wikiwix.com/cache/?url=http://www.planete-sciences.org/espace/publications/techniques/couverture_nacelle.pdf&title=Compte-rendu%20d'exp%C3%A9rience (last verification 18 June 2014)

The results are consistent with the previous analysis, knowing that this experiment is performed in a dry environment. But the crucial importance of this factor (external conduction, linked to meteorological conditions) is not mentioned.

**Critique of instructions for use found on emergency blankets**

Students will easily find on the internet numerous announcements for emergency blankets. They might be asked to analyse their contents. It is noteworthy that these instructions most often seem to rely only on the story of “heat” that is more or less reflected. When a heat source is external, like a fire or the Sun, this line of reasoning leads to a correct idea: the highly reflective side should be put outside. But in case of a risk of hypothermia, nothing is said about the two different choices to be made, depending on meteorological factors: it is currently advised to put the silver side inside, which is appropriate only in case of windy and wet weather.

**Links with other situations**

**Surface state, emissivity and Infra Red images**

With the development of thermal cameras, it might be thought that such cameras measure the temperature of bodies whatever their surface state (colour, roughness). Hence, many IR pictures are presented as if we were directly “seeing” (be it in false colours) the temperature. Additionally, IR images may suggest spectacular effects, due to the transparency in IR of
certain materials (black plastic), and are therefore very attractive. Hence a risk of hasty interpretations.

The work suggested here about emergency blankets highlights the importance of surface state in the radiation emitted by a body due to its own temperature. It may therefore contribute to facilitating an appropriate interpretation of IR images.

**Steady states or quasi stationary changes: systemic analysis V/s linear causal reasoning**

The difficulties that students may feel with the topic of survival blanket are very close to those raised by green house effect. There also, the “story” of a radiation is commonly used as a basis for the analysis. It is currently said that the incoming radiation is trapped by the Earth or by the green house, with no concern about a then inevitable divergence. The co-occurrence of several power fluxes is currently ignored and the explicative diagrams are read from left to right (then, then, then ...). This causes important difficulties when it comes to interpreting, for instance, the fact that the flux emitted by the ground is larger than the incoming flux (Colin 2011).

In this sense, the topic of the survival blanket provides an opportunity to work about ideas that are fruitful when interpreting steady states of disequilibrium or quasi static changes, particularly with fluxes in parallel. With fluxes which are increasing functions of a difference in temperature, with or without a linear approximation, the mechanism for thermal regulation can be analysed in a way close to that discussed here.

**References**


**Appendix 1. Absorptivity, emissivity and Stephan’s law (after Besson 2009)**

For an incoming radiation $R$ having a frequency spectrum $\phi_i(\nu)$ and intensity

$$\Phi_i = \int_0^\infty \phi_i(\nu) d\nu,$$

the intensity absorbed by a body surface will be

$$\int_0^\infty a(\nu)\phi_i(\nu) d\nu = a_r \Phi_i,$$

where $a_r$ represents global absorptivity for the entire considered radiation $R$, and it is equal to the mean value, weighted with $\phi_i(\nu)$, of the absorptivities $a(\nu)$ relative to all frequencies.

The total energy emitted per area and time unit (intensity) is

$$\Phi_e = \int_0^\infty e(\nu)B(\nu, T) d\nu = e_t \sigma T^4,$$

where $e_t$ is the mean value, weighted with the Planck function $B(\nu, T)$, of the surface emissivity $e(\nu)$, $T$ is the absolute temperature and $\sigma$ the Stephan constant.

$a(\nu) = e(\nu)$ for every $\nu$, these coefficients can strongly vary with the frequency.

Black body: $a(\nu) = e(\nu) = 1$ for every $\nu$ and $\Phi_e = \sigma T^4$. 


Appendix 2. A simple way – a variant of the so-called “Leslie cube” – to recall the importance of surface state in emissivity of a body: a boiler filled with water, various materials stuck on the external surface, to be used with an infrared thermometer.

Measures of the respective emitted powers, indicated with equivalent temperature (IR thermometer) when the water in the boiler is boiling:

<table>
<thead>
<tr>
<th>Number displayed by the “thermometer”</th>
<th>Bare metal</th>
<th>Transparent tape</th>
<th>Black electric tape</th>
<th>Emergency blanket “Silver” outside</th>
<th>Emergency blanket “Gold” outside</th>
</tr>
</thead>
<tbody>
<tr>
<td>Situation: Boiling water in a boiler</td>
<td>38 °C</td>
<td>95 °C</td>
<td>96 °C</td>
<td>27 °C</td>
<td>38 °C</td>
</tr>
</tbody>
</table>

Appendix 3. Linear model for heat transfer, according to Volmer (2009, pp. 1082-1083)

“Non linear radiative heat transfer in cooling processes of objects with small Biot numbers does lead to systematic deviations from simple exponential cooling curves. Three related questions were investigated theoretically as well as experimentally: (i) what is the magnitude of deviations? (ii) is it possible to define a range of validity for Newton’s law of cooling? And (iii) can these deviations be easily observed experimentally?

The first and second questions are connected to each other. Theoretical studies which assumed a constant convective heat transfer revealed that the magnitude of the deviations does sensitively depend on the ratio between convective and radiative heat-transfer rates. If radiation dominates, deviations from Newton’s law become obvious already at low temperature differences, of say 30 K. If, however, convection dominates over the radiative heat transfer, Newton’s law may be valid for a much larger range of temperature differences of 500 K (and may be more). In conclusion, there is no single limit of validity, rather it is necessary to discuss the relative contributions of convective and radiative heat-transfer rates.

Therefore, it is easily possible that Newton’s law may be observed for temperature differences as high as, e.g. 200 K, in particular, since in some experimental studies fans were used to achieve high convective heat transfer. It is also logical that in many low temperature experiments like with water in flasks, cans, or bottles and $\Delta T < 50$ K, it is found to be a very good approximation.

However, it is also easily possible to experimentally detect deviations from Newton’s law. For the experiments performed for this study with Al cubes, it worked quite well for up to 40
or 50 K. For higher temperature differences, deviations were clearly present. Similarly, the high temperature region was investigated with halogen light bulbs, with which $\Delta T$ up to 300 K could be realized. Here, Newton’s law provided a reasonable approximation up to about $\Delta T = 100$ K, whereas deviations were obvious for larger temperature differences.

In conclusion, Newton’s law of cooling does successfully describe cooling curves in many low temperature applications. At a first glance this is indeed surprising, in particular when considering the fact that even around room temperature, radiative heat loss is of the same order of magnitude as convective heat loss. At a closer look, however, this result was to be expected: the range of validity of Newton’s law does more or less just depend on the ratio of convective to radiative heat transfer.”