Profile optimization of a cultivator shank

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Abstract

Fracture failures are experienced of locally designed cultivator shanks. The local design differs from the proven designs employed internationally in that it possesses a greater global stiffness to cater for the harder South African soil conditions. The local design fails due to excessive stressing at the top bend. Modifications to the standard design in order to improve the durability of the shanks are discussed. The cross-sectional profiles are optimized to lessen and redistribute the stresses in the failure area whilst retaining the global stiffness of the local design. A modelling and optimization procedure is proposed and discussed in this paper, incorporating finite element analyses and numerical optimization techniques. It is shown that the optimized design lessens the stresses dramatically, thus improving the durability of the cultivator shank.

Nomenclature

E	Young's modulus
I_i	second moment of area of section i
EI_i	flexural stiffness of section i
$\mathbf{x}$	vector of design variables
d_i	thickness of section i
r_i	cross-sectional radius of curvature of section i
θ_i	angle subtended by cross-section i
M	bending moment
M_i	max. bending moment in section i
σ_i	max. tensile stress in section i
y_i	max. distance from neutral axis in section i
F	unit force applied at free tip
l	normal distance from horizontal base line
l_i	normal distance to point of max. stress in section i
V_0	free tip displacement of standard design
V	free tip displacement
$f(\mathbf{x})$	cost function
$g_i(\mathbf{x})$	inequality constraint functions
$h_j(\mathbf{x})$	equality constraint functions
s	arc length along shank
S	total length of shank

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Introduction

The load conditions on a cultivator shank, operating in South African soil, are more severe than those experienced elsewhere. For this reason a proven design was arbitrarily modified to introduce a greater global stiffness. In spite of the greater stiffness of the modified design it experiences high concentrated stresses when working hard soil which causes fracture failures.

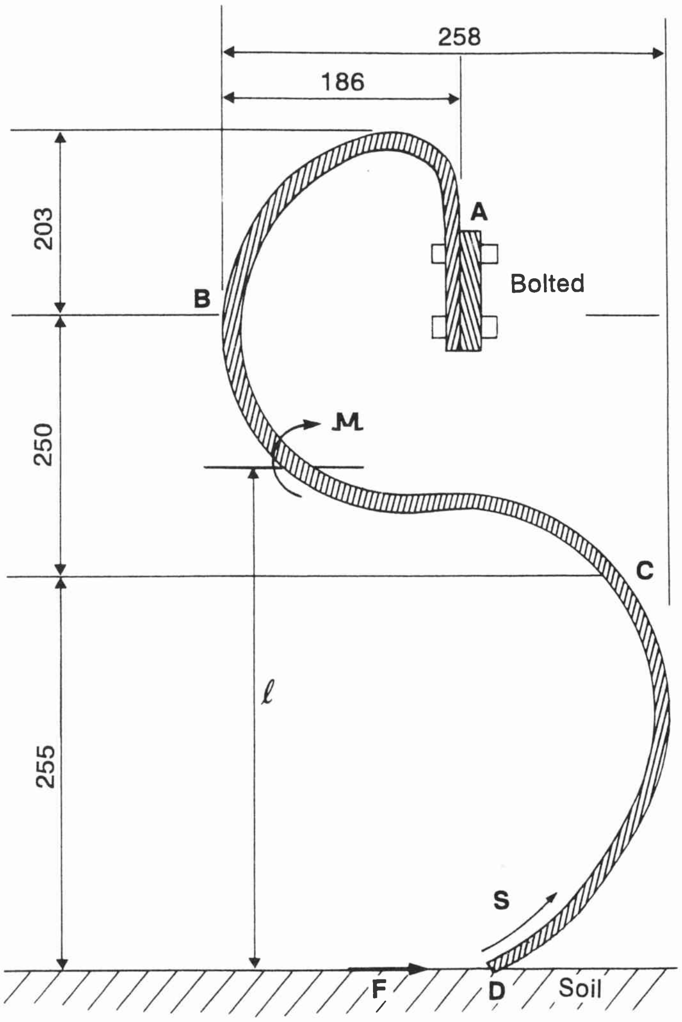
To ensure trouble free operation in South African soil, it is therefore necessary that the shank be redesigned. This paper describes how this may satisfactorily be done through the application of an optimization methodology. As a first step a sensitivity analysis is performed, with the existing local design as reference, to determine the sensitivities of the global stiffness and the maximum stresses in the different sections of the shank, to variations in the physical dimensions. This analysis allows for the definition of meaningful design variables and an appropriate objective function. In particular the objective relates to the maximum stress in the shank under a prescribed unit load at the free tip. A numerical optimization method is then applied to minimize the objective function (maximum stress) with respect to the chosen design variables and subject to prescribed constraints on the global stiffness and design variables.

Sensitivity analysis

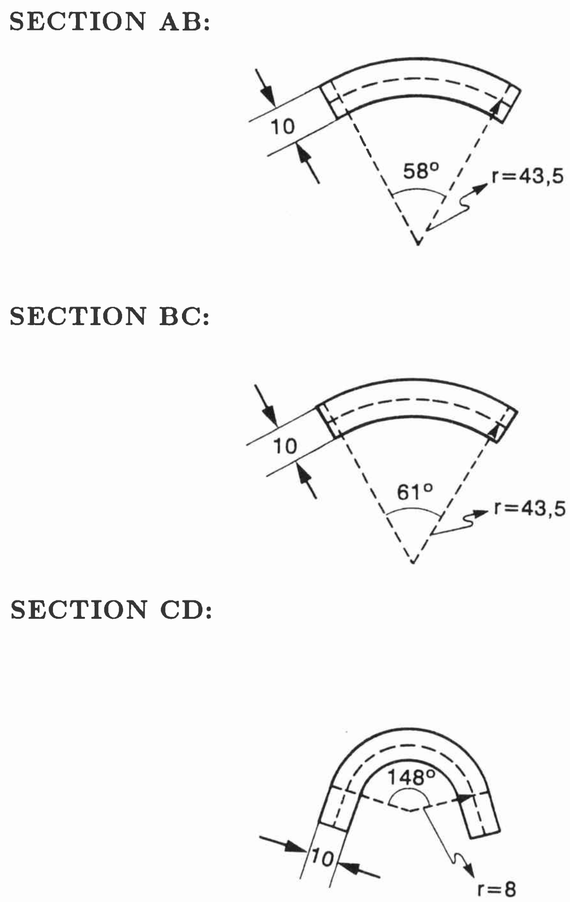
The basic structure of the existing local design is shown in Figures 1 (a) and (b), which represent the side view of the main sections of the shank and the corresponding representative sectional cross-sections, respectively. The standard design fails due to over stressing in the section AB where the maximum bending moment occurs. The design is profiled such that the section AB, in which failure occurs, possesses less stiffness than the lower sections BC and CD, resulting in the major portion of the deformation taking place in section AB. Due to the higher stiffness at the lower sections BC and CD, these parts effectively displace as a single rigid body because most of the translations and rotations occur in the section AB. The contribution of the lower sections, BC and CD in load carrying is accordingly much less than that of section AB.

Any new design must possess sufficient global stiffness, to operate efficiently in South African soil. The shank will not serve its purpose unless it maintains the correct depth. The global stiffness is therefore an important parameter because the ability to maintain the correct depth in the soil depends directly on it. The cross-sectional profiles of sections AB, BC and CD must therefore be redesigned so as to give a more uniform stress distribution whilst still retaining the reference global stiffness.

To further quantify the above observations it is necessary to perform a sensitivity analysis of the standard design. For this purpose the behaviour of the shank may be modelled by means of the finite element method. In particular first order three dimensional beam elements, with sixty-five linear elements along the length of the shank, were used. The model is constrained at the normal clamping position and subjected to a unit load at the free tip in the direction as shown in Figure 1 (a). The free tip is defined as the point where the plough, in contact with the soil, is at its deepest. A load applied at the free tip will result in a maximum bending moment occurring in section AB.



(a)



(b)

Figure 1(a) Side view and (b) representative cross-sectional profiles of the standard cultivator shank. (Dimensions in mm)

Several analyses were performed where the cross-sectional profiles of the different sections, AB, BC and CD, were varied to investigate the sensitivity of the global stiffness and the corresponding maximum tensile stresses σ_{AB}, σ_{BC} and σ_{CD} to different sizes and combinations of sectional profiles. These variations result in corresponding changes in the flexural stiffnesses of the different sections. It is found that a 30% increase in EI_{AB} and EI_{BC}, the flexural stiffnesses of sections AB and BC, respectively, together with a 30% decrease in EI_{CD}, the stiffness of section CD, resulted in a 30% decrease in the maximum stress σ_{AB} in the critical section AB, whilst the global stiffness only decreased by 7%. The results of the different analyses that were performed are listed in Table 1. These results, normalized relative to case 1, which corresponds to the existing design, indicate that it would indeed be possible to achieve a significant improvement of the design by lessening I_{CD} and increasing I_{AB}. This implies that an optimum design exists which minimizes the stress subject to satisfying the prescribed free tip displacement.

Table 1 Sensitivity of maximum sectional stresses and tip displacement to variation in sectional flexural stiffnesses

Case	Normalized flexural stiffnesses			Normalized free tip displacement	Normalized maximum stresses with respect to σ_{AB}		
	EI_{AB}	EI_{BC}	EI_{CD}		σ_{AB}	σ_{BC}	σ_{CD}
1	1	1	1	1	1	0.630	0.200
2	1.3	1.3	0.7	0.93	0.7	0.633	0.407
3	2	1.5	0.7	0.70	0.5	0.850	0.575
4	2	1	1	0.74	0.5	1.267	0.396

Optimization

General optimization problem

The sensitivity analysis of the previous section points to the possibility of obtaining a significantly improved design by the application of a more rigorous optimization procedure. In order to do this the practical engineering problem must be translated into a precisely defined mathematical program of the form

$$\begin{aligned}&\text{minimise}\\ &\qquad f(\mathbf{x}),\quad \mathbf{x}\in\mathcal{R}^n\\ &\text{subject to the inequality constraints}\\ &\qquad g_i(\mathbf{x})\le 0\,,\ i=1,2,\ldots,m\\ &\text{and the equality constraints}\\ &\qquad h_j(\mathbf{x})=0\,,\ j=1,2,\ldots,p\end{aligned} \tag{1}$$

Here $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ is called the vector of *design variables*, $f(\mathbf{x})$ the objective or *cost function* and $g_i(\mathbf{x})$ and $h_j(\mathbf{x})$ the corresponding *constraint functions*.

Design variables

Defining the design variables is a very important step in the modelling and optimization procedure. Insufficient design variables can lead to an over constrained problem. Another factor that must be kept in mind in the modelling of this particular problem, is that the optimum values of the design variables are to be utilized in an actual practical manufacturing process, and must therefore have easily interpretable physical significance.

A sectional profile may be defined by three variables (d,r,θ) as depicted in Figure 2. These variables respectively represent the thickness d of the bar and the radius of curvature r and the angle θ subtended by the curved part of the cross-section. Note that the arc length of the curved part must be less than or equal to the total prescribed centre line length of 50 mm. To allow for the specification of the three different profiles, corresponding to sections AB, BC and CD, a different set of the three variables (d,r,θ) is defined for each. If the variables corresponding to sections AB, BC and CD are respectively denoted by the subscripts $i = 1, 2, 3$ then the chosen vector of design variables $\mathbf{x}$, may be denoted by

$$\mathbf{x} = (x_1, x_2, \ldots, x_9) = (d_1, r_1, \theta_1, d_2, r_2, \theta_2, d_3, r_3, \theta_3)$$

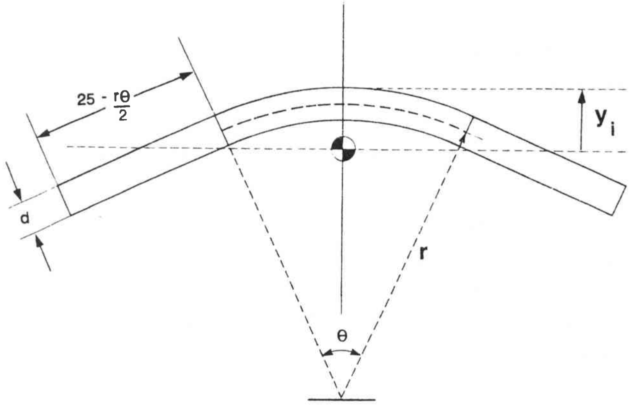


Figure 2 Design variables defining a sectional profile

Cost function

The second moment of area I_i, of each section profile is easily expressed in terms of the corresponding design variables (d_i, r_i, θ_i) of the section. The maximum tensile stress at the point of maximum bending moment is given by

$$\sigma_i = \frac{M_i y_i}{I_i} \tag{2}$$

where $M_i = Fl_i$ is the maximum bending moment in section i (see Figure 1) and y_i is the maximum distance from the neutral axis through the centroid of the cross-section (see Figure 2). The effect of the curvature of the neutral axis is at most 5% and is therefore not taken into account. Since y_i and I_i are clearly both functions of the design variables (d_i, r_i, θ_i), so is σ_i. Having specified values for the set of design variables, $\mathbf{x} = (d_i, r_i, \theta_i)$ $i = 1, 2, 3$; the values of the maximum sectional stresses σ_i in each section i may be determined by using equation (2). This now enables the computation of an appropriate cost or objective function $f(\mathbf{x})$.

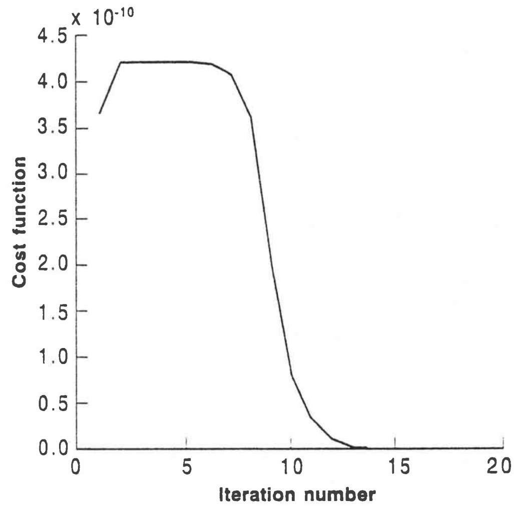


Figure 3 Convergence history of cost function against SQP iteration number

The rationale behind the construction of the cost function is that the maximum stress in the shank is effectively minimized if the maximum stress in all three sections is approximately equal. The cost function is accordingly defined as

$$f(\mathbf{x}) = \frac{(\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2}{\sigma_1^2} \qquad (3)$$

Since the failures experienced on the shank were caused by fatigue, the cost function is based on the von Mises failure theory, which describes the local plastic deformation that leads to fatigue crack initiation.The unconstrained optimum is clearly achieved at the minimum cost function value of zero.

Constraint function

The free tip deflection V, obtained by applying the principle of virtual work, is given by:

$$V = \int_0^S \frac{M^2}{EI} ds \qquad (4)$$

where the integration is performed over the total length S of the shank (see Figure 1). In practice this integration is performed numerically by means of Simpson's rule. Since $I = I_i$ is determined by the specific choice (d_i, r_i, θ_i) for each section i, V is clearly also a function of the design variables (d_i, r_i, θ_i) $i = 1, 2, 3$. From practical experience it was found that efficient operation of the shank is achieved with the overall stiffness of the existing design. It is therefore prescribed that the deflection of the free tip under the unit load must not differ by more than 10% from the deflection V_0 obtained from the finite element analysis of the standard design. The corresponding inequality constraint is accordingly defined by

$$g_1(\mathbf{x}) = \left[\frac{V - V_0}{V_0}\right]^2 - 0.01 \leq 0 \qquad (5)$$

It is also required that the shank be manufactured from an off-the-shelf 50 mm wide spring steel flat bar. This corresponds to the further inequality constraints (see Figure 2):

$$g_{1+i}(\mathbf{x}) = r_i\theta_i - 0.05 \leq 0, \; i = 1, 2, 3 \qquad (6)$$

where θ_i is measured in radians.

For obvious manufacturing reasons it is specified that the thickness of all section be identical, which corresponds to the additional equality constraints

$$\begin{aligned} h_1(\mathbf{x}) &= d_1 - d_2 = 0 \\ h_2(\mathbf{x}) &= d_2 - d_3 = 0 \end{aligned} \qquad (7)$$

Both the cost function and constraint functions are fully described in terms of the design variables $\mathbf{x} = (d_i, r_i, \theta_i)$ $i = 1, 2, 3$. The task of optimization is now to determine a combination of sectional profile parameters which would cause the cost function to attain a value as close as possible to zero whilst satisfying the prescribed constraints, i.e. the optimization problem (1) must be solved. This is done by using the sequential quadratic programming (SQP) algorithm included in the IDESIGN code of Arora.[1] Figure 3 depicts the convergence history showing that convergence to a zero value of the cost function is effectively obtained within twenty iterations of the SQP method.

Results

Note that in this particular description of the sectional profile, in terms of the design variables (d_i, r_i, θ_i), that the greater the angle θ_i, the higher the value of second moment of area I_i for the particular section of interest and vice versa. The existing and optimal designs are listed in Tables 2 and 3, respectively. From a comparison it is clear that there is a dramatic difference between the dimensions of sections AB and CD for the standard and optimal designs. Note also that the thickness d of the optimal design has increased, which is indicative of a more expensive design.

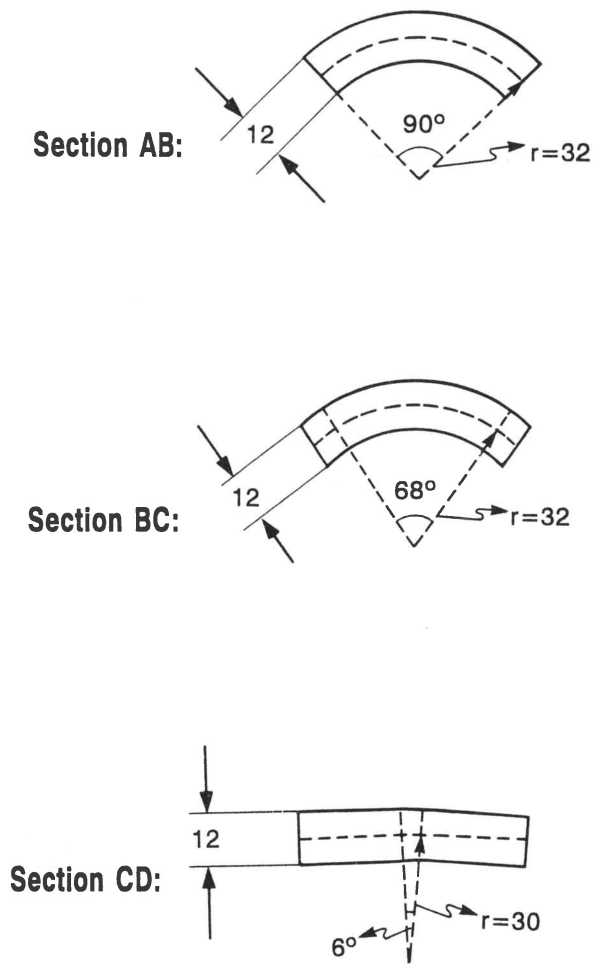


Figure 4 Optimum cross-sectional profiles

Table 2 Existing design

Design variables	Section AB	Section BC	Section CD
r mm	43.5	43.5	8
θ in°	58°	61°	148°
d mm	10	10	10

$$V_0 = 1.975 \times 10^{-5} m$$

Table 3 Optimal design

Design variables	Section AB	Section BC	Section CD
r mm	32	32	30
θ in°	90°	68°	6°
d mm	12	12	12

The optimal design achieves the same global stiffness as the current local design, but, as desired, the maximum stress is reduced by a factor of 2.5 in section AB. The optimum cross-sectional profiles are depicted in Figure 4.

Conclusion

In terms of desirable stress the optimal design gives a remarkable improvement compared to that of the standard design. From a manufacturing point of view, the solution is feasible, since the manufacturer may use off-the-shelf spring steel flat bars. When the values of the angle θ_3 increases, the stiffness of section CD decreases and the stiffness of section AB increases accordingly. This ensures that the material in section CD contributes more to the deformation and load absorption. This new design lessens the stressing of the shank significantly and therefore leads to improved durability.

Reference

[1] Arora JS. IDESIGN Users manual version 3.5.2. *Technical Report No. ODL-89.7, Optimal design code.* University of Iowa, Iowa City, 1989.